Math 19 - Homework

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1. $r_1 = r_2 = 1$, $u_e = 1$

$\lambda g = \mu \frac{d^2 g}{dx^2} - g$ $= (f'(u)|_{u=u_e}) \cdot g$

so $\frac{d^2 g}{dx^2} = \left(\frac{\lambda+1}{\mu}\right) g$. let $c = \frac{\lambda+1}{\mu} > 0$. Then

$g(x) = \alpha e^{\sqrt{c}x} + \beta e^{-\sqrt{c}x}$. Boundary conditions (5) imply $\alpha = \beta = 0$,

so the pair $(g, \lambda)$ doesn't exist, solution $u_e = 1$ is stable.

3. $r_1 = r_2 = 1$, $u_e = -1$

again we find $\lambda g = \mu \frac{d^2 g}{dx^2} - g$, so as in #1 $u_e$ is stable

2. $r_1 = r_2 = 1$, $u_e = 0$

$\lambda g = \mu \frac{d^2 g}{dx^2} + g \Rightarrow \frac{d^2 g}{dx^2} = \left(\frac{\lambda - 1}{\mu}\right)$

when $c = 0$, $g(x) = \alpha + \beta x$. Boundary conditions (5) imply

that $\beta = 0$, so $g(x) = \alpha$ and $\lambda = 1$. We have found a pair

$(g, \lambda) = (\alpha, 1)$ so the solution is unstable.

4. $r_1 = 1$, $r_2 = -1$, $u_e = 0$

we find $\lambda g = \mu \cdot \frac{d^2 g}{dx^2} + g$, so $u_e$ is unstable as in #2

9. $\frac{du}{dt} = 2 \frac{d^2 u}{dx^2} + ru$

a) $u(0) = u(10) = 0$ $\quad 0 = \frac{2 d^2 u}{dx^2} + ru \Rightarrow \frac{d^2 u}{dx^2} = -\frac{r}{2} u$. Let $c = -\frac{r}{2}$.

when $c \geq 0$, get trivial solution.

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9. a) when $c<0$,

$u_c(x) = \alpha\cos(\sqrt{-c}\,x) + \beta\sin(\sqrt{-c}\,x)$

boundary conditions imply $\beta=0$ or $\sin(10\sqrt{-c})=0$, since $\alpha=0$.

$\sin(-10\sqrt{-c})=0 \Rightarrow c = \frac{-n^2\pi^2}{100} \Rightarrow r = \frac{n^2\pi^2}{50}$

c) $u(0)=u(20)=0$

when $c\geq 0$, get trivial solution.

when $c<0$,

$u(x) = \alpha\cos(\sqrt{-c}\,x) + \beta\sin(\sqrt{-c}\,x)$. Again, we find $\alpha=0$ and

$\beta=0$ or $\sin(20\sqrt{-c})=0 \Rightarrow c = \frac{-n^2\pi^2}{400} \Rightarrow r = -2c = \frac{n^2\pi^2}{200}$