Math 19 - Homework

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1. $F(u_e) = a u_e$, $a > 0$. Show eqs 7, 8 can be solved with $u_e \neq 0$ if a can be written as $a = \mu n^2 \pi^2 / L^2$ for $n \in \mathbb{Z}$.

$\mu \cdot \frac{d^2}{dx^2} u_e + F(u_e) = 0$, $\left(\frac{d}{dx} u_e\right)(0) = \left(\frac{d}{dx} u_e\right)(L) = 0$

$\mu \cdot \frac{d^2 u_e}{dx^2} + a \cdot u_e = 0 \Rightarrow \mu \cdot \frac{d^2 u_e}{dx^2} = -a u_e \Rightarrow \frac{d^2 u_e}{dx^2} = \frac{-a}{\mu} \cdot u_e$. Let $c = -\frac{a}{\mu}$.

when $c < 0$, have solutions of the form

$u_e(x) = \alpha \cos(\sqrt{-c}\, x) + \beta \sin(\sqrt{-c}\, x)$

$0 = \frac{du_e}{dx}(0) = \frac{du_e}{dx}(L) \Rightarrow \beta = 0$, $\sqrt{-c} = \frac{n\pi}{L}$ (if $\alpha \neq 0$) (which is the condition we need for $u_e \neq 0$)

so $\sqrt{\frac{a}{\mu}} = \frac{n\pi}{L} \Rightarrow \frac{a}{\mu} = \frac{n^2\pi^2}{L^2} \Rightarrow a = n^2 \cdot \pi^2 \cdot \mu / L^2$

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2. $F(u_e) = -a u_e$, $a>0$. Show no solutions to 7,8 except $u_e=0$.

We have $\dfrac{d^2u_e}{dx^2} = \dfrac{a}{\mu} \cdot u_e$. Let $c = \dfrac{a}{\mu}$. $c>0$ since $a, \mu > 0$.

Then solutions are of the form $u_e = \alpha e^{\sqrt{c}\, x} + \beta e^{-\sqrt{c}\, x}$.

Using boundary condits, we see that $\alpha = \beta = 0$, so there are no nonzero solutions.

3. We only have a nonzero eq. solution if $a = \dfrac{\mu n^2 \pi^2}{L^2}$. It is unlikely to find an a that satisfies this equation in nature.
(where L is the embryo length, μ is the diffusion constant)

5. a) $F(u) = \sin u$
$u_e(x) = \cos 2x$ $\Rightarrow$ $F(u_e(x)) = \sin(\cos(2x))$

$$\left.\frac{dF}{du}\right|_{u=u_e(x)} = \cos(\cos(2x))$$

c) $F(u) = 1+u^2$
$u_e(x) = 1+x^2$ $\Rightarrow$ $F(u_e(x)) = 1+(1+x^2)^2$

$$\left.\frac{dF}{du}\right|_{u=u_e(x)} = 2+2x^2$$