Homework 10

Chapter 15

1. a) $u(t,x) = 2\frac{1}{t^{1/2}} e^{-x^2/4\mu t} + \frac{1}{t^{1/2}} e^{-(x-1)^2/4\mu t}$

$$\frac{du}{dt} = -t^{-3/2} \cdot e^{-x^2/4\mu t} + \frac{2}{t^{1/2}} \cdot e^{-x^2/4\mu t} \cdot \frac{x^2}{4\mu t^2} - \frac{1}{2} t^{-3/2} e^{-(x-1)^2/4\mu t} +$$
$$t^{-1/2} \cdot e^{-(x-1)^2/4\mu t} \cdot \frac{(x-1)^2}{4\mu t^2}$$

$$\frac{d^2u}{dx^2} = 2t^{-1/2} e^{-x^2/4\mu t} \cdot \frac{x^2}{4\mu^2 t^2} + t^{-1/2} e^{-(x-1)^2/4\mu t} \cdot \frac{(x-1)^2}{4\mu^2 t^2} - \frac{1}{\mu t} \cdot t^{-1/2} \cdot e^{-x^2/4\mu t} - \frac{1}{2\mu t} t^{-1/2} \cdot e^{-\frac{(x-1)^2}{4\mu t}}$$

$$\mu \cdot \frac{d^2u}{dx^2} = \frac{x^2}{2\mu} \cdot t^{-5/2} e^{-x^2/4\mu t} + \frac{(x-1)^2}{4\mu} t^{-5/2} \cdot e^{-(x-1)^2/4\mu t} - t^{-3/2} \cdot e^{-x^2/4\mu t} - \frac{1}{2} t^{-3/2} \cdot e^{-(x-1)^2/4\mu t}$$

Simplifying, we get $\frac{d}{dt}u = \mu \frac{d^2}{dx^2} u$

c) $u(t,x) = 2 \cdot t^{-1/2} \cdot e^{-x^2/4\mu t} + 3 \cdot t^{-1/2} \cdot e^{-(x-1)^2/4\mu t}$

$$\frac{du}{dt} = -t^{-3/2} \cdot e^{-x^2/4\mu t} + \frac{x^2}{2\mu} t^{-5/2} \cdot e^{-x^2/4\mu t} - \frac{3}{2} t^{-3/2} \cdot e^{-(x-1)^2/4\mu t} + \frac{3(x-1)^2}{4\mu} \cdot t^{-5/2} \cdot e^{-(x-1)^2/4\mu t}$$

$$\mu \cdot \frac{d^2u}{dx^2} = \frac{x^2}{2\mu} \cdot t^{-5/2} \cdot e^{-x^2/4\mu t} + \frac{3}{4\mu}(x-1)^2 \cdot t^{-5/2} \cdot e^{-(x-1)^2/4\mu t} - t^{-3/2} \cdot e^{-x^2/4\mu t} - \frac{3}{2} t^{-3/2} \cdot e^{-(x-1)^2/4\mu t}$$

2. a) $u(t,x) = 2e^{\lambda t} e^{x\sqrt{\lambda/\mu}} + 3e^{\lambda t} e^{-x\sqrt{\lambda/\mu}}$

$$\frac{du}{dt} = 2\lambda e^{\lambda t} e^{x\sqrt{\lambda/\mu}} + 3\lambda e^{\lambda t} e^{-x\sqrt{\lambda/\mu}}$$

$$\frac{d^2u}{dx^2} = 2\frac{\lambda}{\mu} e^{\lambda t} e^{x\sqrt{\lambda/\mu}} + 3\frac{\lambda}{\mu} e^{\lambda t} e^{-x\sqrt{\lambda/\mu}}$$, simplifying we get:

$$\frac{d}{dt}u = \mu \cdot \frac{d^2u}{dx^2}$$

c) $u = 2e^{\lambda t} e^{x\sqrt{\lambda/\mu}} + 1 - 5x$ $\quad \frac{du}{dt} = 2\lambda e^{\lambda t} e^{x\sqrt{\lambda/\mu}}$ ✓

$$\frac{d^2u}{dx^2} = 2\frac{\lambda}{\mu} e^{\lambda t} e^{x\sqrt{\lambda/\mu}}, \quad \mu\frac{d^2u}{dx^2} = 2\lambda e^{\lambda t} e^{x\sqrt{\lambda/\mu}}$$ ✓ =