MATHEMATICS 191, FALL 2004
MATHEMATICAL PROBABILITY
Assignment #6a

Problems to be discussed in section on Monday, Nov. 9:

All problems are from Grimmett and Stirzaker, 1000 Exercises in Probability. The solutions are all in the book!

Add together the number of letters in your first and last name. If the sum is odd, prepare problems 1,3, and 5. If it is even, prepare 2, 4, and 6.

The first two problems are an attempt to make up for losing a lecture to the Nov. 11 holiday. In this course, you are responsible only for knowing these formulas, which PRP takes for granted. However, ever well-educated mathematician, physicist, or mathematical economist should be able to prove these famous formulas.

1. Starting from Wallis's product

$$\pi/2 = \frac{2/1 * 4/3 * 6/5...}{3/2 * 5/4 * 7/6...} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdots = \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdots}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdots}$$

, proved in the notes, show that for large n,

$$u_{2n} = \frac{(2n)!}{2^{2n}(n!)^2} \approx \frac{1}{\sqrt{\pi n}}$$

2. By approximating $log(n!)$ by an integral as in outline 6, #11, argue for Stirling's approximation

$$n! \approx K n^n e^{-n} \sqrt{n}$$

and show that if you set $K = \sqrt{2\pi}$, you again get

$$u_{2n} = \frac{(2n)!}{2^{2n}(n!)^2} \approx \frac{1}{\sqrt{\pi n}}$$

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3. Section 3.11 , problem 39a. (random walk with jump from level n to a random level)

4. Section 3.11, problem 23 (random walk with reflecting barrier. Every time you go broke, the casino gives you another dollar to bet).

5. Section 3.11, problem 28. (arc sine law for maxima)

6. Section 3.11, problem 29. (levels visited exactly once)

Since Thursday, Nov. 11 is a holiday, there are no problems due that day! An assignment due Tuesday, Nov. 16 will be distributed on Nov. 9.

Sections will NOT meet on Nov. 15, but the CAs will hold reviews on Wednesday, Nov. 17 for the quiz on Nov. 18.