MATHEMATICS 191, FALL 2004
MATHEMATICAL PROBABILITY
Assignment #4

Problems to be discussed in section on October 18:
All problems are from Grimmett and Stirzaker, 1000 Exercises in Probability. The solutions are all in the book!

Add together the number of letters in your first and last name. If the sum is odd, prepare problems 1, 3, and 5. If it is even, prepare 2, 4, and 6.

1. Section 2.1, problem 4. (weighted average of distribution functions)
2. Section 2.7 , problem 3a. Do $X + Y$ only. This is surprisingly difficult because of the restriction to countable unions. (sum of random variables)
3. Section 2.2 , problem 1. (avoiding embarrassment)
4. Section 2.2, problem 3. (estimating a distribution function)
5. Section 2.7, problem 12. (loaded dice)
6. Section 1.8, problem 21. Get a numerical answer for $p = \frac{1}{2}, r = 4, s = 3$. (runs of heads and tails)

Problems to be handed in on Thursday, October 21:

1. Suppose that the random variable X is uniformly distributed in the interval (1, 27] and that the random variable Y is given by the function $Y = X^{\frac{1}{3}}$.
   (a) What is the probability of the event $2 < Y \leq 3$?
   (b) Determine the distribution function $F_Y(t)$ and a density function $f_Y(t)$ on the interval $1 < Y \leq 3$. Sketch graphs of both. Show that $F_Y(t)$ is consistent with your answer to part a.
2. Using the same approach as in section 2.2, prove directly that
$$\mathbb{P}(\frac{1}{n}S_n \leq p - \epsilon) \leq e^{-\frac{1}{4}n\epsilon^2}$$
for $\epsilon > 0$. You may adopt the same proof strategy as in the book and in lecture, but you may not assume the result that was proved. In other words, start with a lower limit of 0 and an upper limit less than n. Various signs will need to be changed from the original proof.
3. A baseball playoff series ends when either team wins 4 games. Team 1 has probability p of winning each game, independently of the other games. Let the random variable X be the number of games in the series. Calculate the probability mass function for X for the case $p = 0.6$, and determine its expectation and variance.

4. In the final game of the World Series, a baseball manager has n pitchers available in his bullpen. Of these, r will throw nothing but strikes, while the other $n - r$ will throw nothing but balls. Nobody knows which is which, of course! The manager brings pitchers into the game in a random order. The random variable X is the number of pitchers he must try before finding one who throws strikes. Find the probability mass function for X, and calculate its expectation. Assume $r > 0$, though in real life this is not necessarily so.

5. Contestants A, B, and C are participating in a home-run hitting contest. They go in the order ABCABCA.... Each participant has probability $p = \frac{1}{3}$ of hitting a home run on each attempt.

   (a) What is the probability that both A and B hit home runs before C does?

   (b) What is the probability that the third home run of the contest, but neither the first nor the second, is hit by C?

6. n "Iraqi most wanted" prisoners are still languishing in Guantanamo. They persuade a judge that depriving them of playing cards is a human-rights violation, and so they are given the n U.S. military cards that have their pictures. Let h_n be the number of ways of dealing these cards, one to each prisoner, so that no prisoner gets his own card.

   Now one more prisoner arrives, then another, and their cards are added to the deck. Analyze the situation to find a formula for h_{n+2} in terms of h_{n+1} and h_n. By solving this recurrence relation, get an alternative derivation of the solution to the matching problem.