MATHEMATICS 191, FALL 2004
MATHEMATICAL PROBABILITY
Assignment #2

Problems to be discussed in section:
All problems are from Grimmett and Stirzaker, 1000 Exercises in Probability. The solutions are all in the book!

Add together the number of letters in your first and last name. If the sum is odd, prepare problems 1, 3, and 5. If it is even, prepare 2, 4, and 6.

1. Section 1.4, problem 3, excluding part (v).
2. Section 1.4, problem 4.
3. Section 1.5 , problem 2.
4. Section 1.5, problem 5.
5. Section 1.8, problem 12.
6. Section 1.8, problem 24.

Problems to be handed in on Thursday, Oct. 7 :

1. Boxcar Bob owns three dice. Two of them are unloaded, but the third has $p = 1/2$ for a 6, and $p = 1/10$ for the numbers 1 through 5.
   Event B is that he rolls a randomly chosen die three times, and a 6 appears precisely once.
   Event A is that Bob is using the loaded die.

   (a) Calculate $\mathbb{P}(A \cap B), \mathbb{P}(A^c \cap B)$, and $\mathbb{P}(B)$.

   (b) Given event B, what is the conditional probability that Bob is using the loaded die?

2. The third problem from the "Bayesian Bible" attached to outline 3.

3. The last problem from the "Bayesian Bible" attached to outline 3.

4. In the admissions office at Monty Hall University there are four interviewers. Three of them, Fl, F2, and F3, are friendly, while the fourth, U, is unfriendly. Every morning the Dean of Admissions assigns them randomly to offices 1, 2, 3, and 4, with an equal probability for each possible assignment. A student arrives for an interview and is asked to select which office he wants to be interviewed in. He chooses office 1 and learns that the interviewer in there is busy for the next half hour. "While you are waiting," says the Dean to the student, "I would like you to meet one of our friendly interviewers. From offices 2, 3, and 4, I will choose the lowest-numbered friendly interviewer." He opens the door of office 2 and introduces the student to an interviewer.
   This is Event B – the lowest-numbered available friendly interviewer was in office 2.
   Event A is that office 1 contains the unfriendly interviewer.

   (a) Enumerate all the ways of assigning interviewers to offices that lead to Event B. Assign a probability to each, and show that the sum of these probabilities equals the probability of Event B.

   (b) Given that Event B has occurred, determine the conditional probability of event A.

   (c) What is the probability that the friendly interviewer to whom the student was introduced by the Dean was F1?

   An ambitious preschool, whose aim is to prepare toddlers for Harvard, has 8 students, 4 boys and 4 girls. Following the example of Harvard, it divides its students randomly into two groups of 4, which it names "Lowell House" and "Eliot House."

   (a) What is the probability P_4 that all four boys end up in Eliot House?

(b) What is the probability P_3 that three boys and one girl end up in Eliot House, with the other boy and three girls in Lowell House?

(c) What is the probability P_2 that optimal diversity is achieved, with two boys in Eliot House and two in Lowell House?

(d) Verify that the sum of probabilities for all the ways of dividing the class between the houses is correct.

(e) A new teacher, assigned to Eliot House, arrives at the school and asks to meet a randomly chosen student from the house. It is a girl (event B). What is the conditional probability that there are more girls than boys in Eliot House?

(f) The teacher now asks to meet a randomly chosen student from Lowell house. It is a boy (event C). What is the conditional probability (conditioned on $B \cap C$) that there are more girls than boys in Eliot House?

5. Poker novice Jane picks up her five cards and asks "What did you say the probability was for event A (no two cards of the same rank)?" Veteran Betty tells her. Jane then says "Well, event B (all four suits represented in the hand) has just occurred for me. Is that worth anything?" Betty says, "No, but given that, do you want to know the conditional probability for A, which I'm planning to use when I bet against you?"
Calculate $P(A)$, $P(B)$, $P(A \cap B)$, and $P(A|B)$. You'll want a calculator.

6. (a) For the example of Simpson's paradox on p.19 of PRP, assume that probabilities are proportional to observed frequencies and do a numerical check of the formula

$$\mathbb{P}(A|B) = \mathbb{P}(A|(B \cap C))\mathbb{P}(C|B) + \mathbb{P}(A|(B \cap C^c))\mathbb{P}(C^c|B)$$

and the corresponding formula involving A^c. You'll have to work out the various conditional probabilities.

(b) Invent an example of Simpson's paradox to show that "an inferior product is better." It should not involve drug tests, and the test groups should not be men and women. Example: you are a sports agent. Your baseball player hits worse, against both lefties and righties, than a competing player, but you rig a test that gets him a higher batting average.