MATHEMATICS 191, FALL 2004
MATHEMATICAL PROBABILITY
Assignment #1

Problems to be discussed in section on Sept. 26, 27 or 28: All problems are from Grimmett and Stirzaker, 1000 Exercises in Probability. The solutions are all in the book!

Add together the number of letters in your first and last name, ignoring spaces and hyphens. If the sum is odd, prepare problems 1, 3, and 5. If it is even, prepare 2, 4, and 6.

1. Section 1.2, problem 1. (De Morgan's laws)
2. Section 1.3, problem 2. (Murphy's Law)
3. Section 1.8, problem 3. (σ-fields)
4. Section 1.2, problem 4. (Subsets and σ-fields)
5. Section 1.3, problem 4. Just do the Vice-Chancellors example, since the proof was done in lecture.
6. Section 1.8, problem 13. Another way to solve the same Vice-Chancellors problem)

Problems to be handed in at class on Thursday, Sept. 30:

1. Prove that any set of disjoint intervals of positive length is countable, but that the set of all intervals of positive length is uncountable.
2. (a) Problem 7 on p. 505 of Apostol. This extends the proof done in lecture to the case of sets that are not disjoint.
   (b) You have agreed to test every problem on the Harvard online math placement tests. It then turns out that there are a countably infinite number of topics and a countably infinite number of questions for each of them. To make matters worse, some questions are repeated under more than one topic. Using the proof strategy from Apostol, describe a strategy for establishing an order of testing the questions that guarantees that each question will be tested once and only once.
3. EP, page 49, problem 20.
4. Suppose that poker were played with hands of six cards instead of five. Calculate the number of hands of each of the following types, and rank the hands in terms of their probability from least probable to most probable. (Use Mathematica to do the arithmetic if you wish to.)

- “Full hotel”: four cards of one rank, two of another.
- ”Four of a kind”: four cards of one rank, others of different ranks.
- “Full house:three cards of one rank, two of a second, third of a different rank.
- “Two triples”: three cards of each of two ranks.
- “Three pairs”: two cards of each of three ranks.
- “Flush”: all six cards of the same suit.

5. A card player has a hand of 13 cards, of which 7 are red and 6 are black.

   (a) If she selects a pair of cards at random from the hand, what is the probability that they are both red? Both black? Of different colors?

   (b) If, instead, she selects a card at random from the hand, notes its color, replaces it, and again selects a card at random, what is the probability that both selected cards were red? Both black? Of different colors?

6. The year is 2202, and Earth can support its large population only by having billions of people on cruise ships. Marcia Weis, the bridge director for Megaprincess Lines, is preparing a duplicate bridge tournament for about 10 million players. In the opening deal, she arranges for the North-South pairs all to have the same hands, including 9 spades – all but the queeen, 4, 3, and 2. She then distributes the remaining 26 cards (4 spades and 22 others) in every possible way between East and West. Thus every declarer, in trying to guess how the missing cards are divided between East and West, will know that each possible outcome will occur precisely once on a cruise somewhere.

   In what follows, let N = 22!/ (13!)(11!) = 4522 and express all answers as a multiple of N.

   (a) How many different East hands of 13 cards can Marcia prepare?

   (b) In how many of these hands does East have all four of the missing spades?

   (c) In how many of these hands does East have all of the missing spades except the queen?

   (d) In how many of these hands does East have precisely three of the missing spades?

   (e) In how many of these hands does East have precisely two of the missing spades? (As a check, add together this number, twice the answer to part b, and twice the answer to part d. This should agree with the answer to part a).

(f) Since Princess passengers have been taught the maxim "Eight ever, nine never" at the mandatory lifeboat drill, they will all play this deal the same way, by leading the ace and king of spades and hoping that East or West will be forced to play the queen. This approach will succeed if East and West each has two spades or if either East or West has only the queen while the other has the remaining three spades. In how many of the hands will the approach succeed? What is the probability of success?