Math 191 Notes, 2003 September 23

Given an increasing sequence of sets $A_1 \subseteq A_2 \subseteq A_3 \ldots$,

$$A = \bigcup_{i=1}^{\infty} A_i$$

Inclusion-Exclusion

$$\begin{aligned}\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(A \cap B) \\ -\mathbb{P}(A \cap C) - \mathbb{P}(B \cap C) + \mathbb{P}(A \cap B \cap C)\end{aligned}$$

Poker

All hands $\binom{52}{5} = 2568960$

4 of a kind $13 \cdot 48 = 624$

Full house $13 \cdot 12 \cdot 4 \cdot 6 = 3744$

3 of a kind $13 \cdot 4 \cdot 48 \cdot 44 = 109824$

Bridge

See outline and Durango Bill's website.

Conditional Probability

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

This is sometimes known as Bayes' rule, given $\mathbb{P}(B) > 0$.
Example: Bearded Ben, explosives in shoes. A is "explosive shoes", B is "bearded", $\mathbb{P}(B|A) = 0.6$, $\mathbb{P}(B|A^c) = 0.05$, $\mathbb{P}(A) = 0.2$.
Lemma: $A = (A \cap B) \cup (A \cap B^c)$ disjoint, $\mathbb{P}(A) = \mathbb{P}(A \cap B) + \mathbb{P}(A \cap B^c)$, $\mathbb{P}(A) = \mathbb{P}(A|B)P(B) + P(A|B^c)B^c$.
Continued: $\mathbb{P}(B) = \mathbb{P}(B|A)\mathbb{P}(A) + \mathbb{P}(B|A^c)\mathbb{P}(A^c) = 0.16$.
Example: Probability of two-child family which has at least one boy of having the other child being a boy is 1/3.
Example: Monty Hall problem.