Math 191 Notes, 2003 November 6

Normal Distribution continued

$$N(0,1) f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2}$$

has expectation 0, variance 1.

$$Y = \frac{X-\mu}{\sigma} \qquad X = \mu + \sigma Y$$

$$\mathbb{E}(X) = \mu + \sigma \mathbb{E}(Y) = \mu$$

$$\mathrm{var}(X) = 0 + \sigma^2(1)$$

So then,

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma}} N(\mu, \sigma^2)$$

has expectation μ, variance σ^2.

An example of this is voice recognition software. Say one has measured the signal in various frequency bands. Then you assume (incorrectly) that this is a normal distribution and match. Works pretty well. This is used because it is easy to calculate.

Gamma distribution

$$f(x) = \frac{\lambda^t}{\Gamma(t)} x^{t-1} e^{-\lambda x}$$

where $\Gamma(t+1) = t!$ if t is an integer. An example of this is the physics of gases: the "most likely" velocity vector is $< 0,0,0 >$, but the most likely speed is a Maxwell distribution where $t = 3$.

Cauchy distribution

Recall the "bad compass" example where one gives a contestant a bad compass and sends him off to get drinking water.

$$f(x) = \frac{1}{\pi(1+x^2)}$$

We'll generate on a computer an angle that is uniform in $(-\pi/2, \pi/2)$. If it's positive, you pay me the tangent. If it's negative, I pay you the tangent.

$$\mathrm{var}(X) = \int_{-\infty}^{\infty} x^2 f(x) dx = \int_{-\infty}^{\infty} \frac{x^2 dx}{\pi(1+x^2)} \text{ diverges badly}$$

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} \frac{xdx}{\pi(1+x^2)}$$

is also undefined in this class because if we look only at the positive or negative values, we can see that they diverge.

$$\mathbb{E}(X) = \lim_{N\to\infty} \lim_{M\to\infty} \int_{-M}^{N} \frac{xdx}{\pi(1+x^2)}$$

This formal definition shows us that if we ever bound it from below, the expectation is infinite. This is the notorious distribution.

Homework # 4

$$U = g(X) \qquad V = h(y)$$

$$\begin{aligned} k(u,v) &= \mathbb{P}(U \le u, V \le v) \\ &= \mathbb{P}(X \in B_X(u), Y \in B_Y(v)) \end{aligned}$$

where we define u to be a countable union on the x-axis, and v to be a countable union on the y-axis.

$$\int_{\text{rectangles}} f(x,y)dx\,dy = \int_{\text{rectangles}} f_X(x) f_Y(y) dx\,dy$$