Math 191 Notes, 2003 September 25

Typo on new problems (homework 2), Last problem, last word change to “uncountable”!

Independence

A and B are independent if $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$. Almost equivalent. $\mathbb{P}(A|B) = P(A)$. Independent is NOT the same as disjoint!

Generate a random selection from $0, 1, 2, 3$ Generate another one using different random numbers. Generate x, y form $4x + y \in 0, \ldots, 15$.

Being careful about independence:

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$$

$A \cap B$), A, and B must all be in the same σ-field. If they aren't in the same σ-field (say coins and dice), then we need to fix that up. Use language such as “Coin = H, die = anything”. Then everything is pretty much fixed.

What is the number of “2 pair” poker hands?

$$\binom{13}{2} \cdot \binom{4}{2}^2 \cdot 44$$

Bernoulli trials

Experiment: “Success” S with probability p, “Failure” F with probability $q = 1 - p$.
Compound Experiment: n independent repetition, probability of k successes.
Any specific sequence of successes and failures has probability $p^k q^{n-k}$. Total probability $\binom{n}{k} p^k q^{n-k}$.

Infinite sample space

What is the probability that a “6” appears for the first time on roll k?

$$\mathbb{P}(1) = 1/6$$

$$\mathbb{P}(2) = (5/6)(1/6)$$

$$\mathbb{P}(3) = (5/6)^2(1/6)$$

Simpson's paradox

Two drugs tried out on men and women. successes and failures:

Women		Men	
I	II	I	II
200	10	19	1000
1800	190	1	1000

Women		Men	
I	II	I	II
.05	.00	.05	.3
.45	.05	0	.1

Event A = success
Event B = drug I was taken
Event C = subject was a woman

Drug I is better than Drug II for a woman, means that you're calculating $\mathbb{P}(A|B \cap C)$. Compare that with the probability $\mathbb{P}(A|B^c \cap C)$. Much higher.

$$\mathbb{P}(A|B \cap C) > \mathbb{P}(A|B^c \cap C)$$

$$\mathbb{P}(A|B \cap C^c) > \mathbb{P}(A|B^c \cap C^c)$$

$$\mathbb{P}(A|B) < \mathbb{P}(A|B^c)$$

Let's take a look at the numerical values of all the terms here.

$$\mathbb{P}(A|B \cap C) = \frac{\mathbb{P}(A \cap B \cap C)}{P(B \cap C)} = \frac{0.05}{0.5} = \frac{1}{10}$$

$$\mathbb{P}(A|B^c \cap C) = 0$$

$$\mathbb{P}(A|B \cap C^c) = 1$$

$$\mathbb{P}(A|B^c \cap C^c) = \frac{3}{4}$$

$$\mathbb{P}(A|B) = .1/.55 = \frac{2}{11} = \frac{1}{10}\frac{10}{11} + 1 \cdot \frac{1}{11}$$

$$\mathbb{P}(A|B^c) = .3/.45 = \frac{2}{3} = 0 \cdot \frac{1}{9} + \frac{3}{4} \cdot \frac{8}{9}$$

The reason these are so misleading is because we are doing a WEIGHTED AVERAGE that emphasizes the wrong subsets.

Conditional Probabilities

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(A \cap B \cap C) + \mathbb{P}(A \cap B \cap C^c)}{\mathbb{P}(B)}$$

$$= \frac{\mathbb{P}(A|B \cap C)\mathbb{P}(B \cap C) + \mathbb{P}(A|B \cap C^c)\mathbb{P}(B|C^c)}{\mathbb{P}(B)}$$

$$= \frac{\mathbb{P}(A|B \cap C)\mathbb{P}(C|B)\mathbb{P}(B) + \mathbb{P}(A|B \cap C)\mathbb{P}(C^c|B)\mathbb{P}(B)}{\mathbb{P}(B)}$$

$$= \mathbb{P}(A|B \cap C)\mathbb{P}(C|B) + \mathbb{P}(A|B \cap C)\mathbb{P}(C^c|B)$$

What case is drug II better? "I know someone who took drug II and I know someone who took drug I." "I'll bet that the person who took drug II was cured."

Assignment – Find the most interesting Simpson's paradox example on the web.