Math 191 Notes, 2003 September 16

Paul Bamberg Apostol introduction to probability theory – Coursepack?
SC Office hours: 2:30-3:30pm Tuesday, Science Center
Normal hours Wednesday morning, Quincy, M&W evenings, Quincy

Monte Carlo

3 Envelopes - 1 chosen at random, another (not the correct one) is revealed, switch? List permutations:
1 2 3
P H h
P h H
h P H
H P h
H h P
h H P

Functions

Given a function $f : A \to B$ (maps all elements of A into a single element of B), injective, surjective, bijective properties. If bijective, then there is the same number of elements in each set. Cardinality, trivial with finite sets, non-trivial for infinite sets.

Infinite sets

If you can establish a bijection between set of interest and natural numbers, then countably infinite. Also can be done algorithmically in a computer program that generates each item once and only once. Countable means either finite or countably infinite.

- **2-element subsets of $\{1, 2, 3, \ldots\}$.** "Sum approach", "Largest element approach"
- **Positive rational numbers.** "Diagonal" proof.
  1/1 2/1 3/1 4/1
  1/2 2/2 3/2 4/2
  1/3 2/3 3/3 4/3
  We can omit the ones that have already been listed ($1/1 = 2/2$), or we can just count them all and use the fact that it is surjective to say that this is at least countable.
- **Properties in Apostol about countability.**

- Every subset of a countable set is countable.
  Pf. Set S, with subset A. Let S be $x_1, x_2, x_3, \ldots$, circle the ones in A.
- Intersection of countable sets is countable.
  Pf.
- Finite Cartesian product of countables is countable.
  Pf. $P = \{1, 2, 3, \ldots\}$, $PxP = \{(1,1), (1,2), (2,1), (1,3), (2,2), (3,1)\}$ or you can take Cartesian product and map to $2^n 3^m$, a subset of the natural numbers, which is countable. Expand to finite number of countable sets using induction. $(m, n, r, \ldots, s, t) \rightarrow \ldots$.
- Countable union of countable sets is countable.
  Pf. The set $\{B_1, B_2, B_3, B_4, \ldots\}$ are countable. Let the ith element in set j be $B_{i,j}$. List them in matrix form and use diagonal argument.

- **Uncountable sets?**
  - Cantor. The set of reals satisfying $0 < x < 1$ is uncountable.
    Pf. By contradiction. Assume we can make a list of all of them. Look at diagonal, replace 1 by 2, replace anything else by 1. Write this as a decimal, it cannot be on the list because it disagrees with everything in at least one place.
  - Constructing a subset not in a list (extension of previous method).
    For item n, flip element n to be or not be in subset.
  - What is a "random real number"?
  - http://www.longbets.org/
  - Mile of border to defend. Start out at center, make a 1/3 length step in 1/2 time each time.