

MATHEMATICS 191, FALL 2003
MATHEMATICAL PROBABILITY
Assignment #7

Problems to be discussed in section: All problems are from Grimmett and Stirzaker, 1000 Exercises in Probability. The solutions are all in the book!

Add together the number of letters in your first and last name. If the sum is odd, prepare problems 1,3, and 5. If it is even, prepare 2, 4, and 6.

1. Section 4.14, problem 2.
2. Section 4.1 , problem 1b.
3. Section 4.3 , problem 3.
4. Section 4.4, problem 3.
5. Section 4.14, problem 1a.
6. Section 4.14, problem 1b.

Problems to be handed in on Thursday, November 6:

1. (This is an easy but instructive problem, based on the last two section problems from last week. When we do characteristic functions this "problem of the points" will arise yet again.)

Legendary baseball manager Bill Veeck once sent a midget to the plate in hopes that he could draw a walk. The midget had instructions to take pitches until he walked or struck out, and of course because of his small strike zone, the probability p that the pitcher would throw a ball was fairly high. Calculate, as a function of p , the probability that the midget will walk: that he will receive 4 balls before 3 strikes. Although there is no closed-form solution to this problem, you can get the answer as a sum of three terms in two different ways, and both should lead to the same polynomial in p .

- (a) Add together the probability for the midget to walk on 4, 5, or 6 pitches, taking 0, 1, or 2 strikes before the final ball.
- (b) Assume that the pitcher throws 6 pitches, at which point the midget has struck out if he has not walked. Add together the probability for 4, 5, or 6 of them to be balls.

If you are after Grady Little's job, you may want to evaluate the answer for a few values of p , but you are not required to. If you do, consider using Mathematica.

2. Let f_{2n} denote the probability that a symmetric random walk returns to the origin (level 0) for the first time after $2n$ steps. Let u_{2n} denote the probability that a symmetric random walk returns to the origin (level 0) after $2n$ steps. As was proved in class, u_{2n} is also equal to the probability of a variety of other events.
 - (a) Show that $f_{2n} = u_{2n-2} - u_{2n}$ by moving the last term to the left-hand side and interpreting the equation in terms of a union of disjoint events.
 - (b) From the well-known formula for u_{2n} , derive the formula for f_{2n} .
3. Suppose that random variable X has the distribution function $F(x) = \frac{2}{\pi} \arcsin \sqrt{x}$. Calculate its expectation (obvious by symmetry, but do the integral) and its variance.
4. Prove theorem 3 on page 92 of G&S. Invent an example where g and h are both discontinuous but the result still holds.
5. X is a random variable that is uniformly distributed on $[0, \frac{\pi}{2}]$. For each of the following random variables, determine the distribution function and the density function, and sketch a graph of each.
 - (a) $Y = \sin^2 X$.
 - (b) $Z = \tan \frac{X}{2}$.