MATHEMATICS 191, FALL 2003
MATHEMATICAL PROBABILITY
Assignment #5

Problems to be discussed in section:
All problems are from Grimmett and Stirzaker, 1000 Exercises in Probability. The solutions are all in the book!

Add together the number of letters in your first and last name. If the sum is odd, prepare problems 1,3, and 5. If it is even, prepare 2, 4, and 6.

1. Section 3.3, problem 3a.
2. Section 3.3 , problem 8.
3. Section 3.6 , problem 1.
4. Section 3.11, problem 7.
5. Section 3.11, problem 13a and b(first method).
6. Section 3.11, problem 13b(second and third methods).

Problems to be handed in on Thursday, October 23:

1. A necklace contains 23 pearls strung on a circular wire. 16 of them are worthless imitations, but the other 7 are worth \$1000 each. Use the "probabilistic method" or an equivalent counting argument to show that if you are allowed to choose a sequence of 10 consecutive pearls, you can always find one that is worth at least \$4000. Can you also be sure of finding some sequence that is worth \$2000 or less?
2. In class, the mean and variance for the negative binomial distribution (G&S, page 61) were computed by considering a sum of r random variables, each with the geometric distribution. Rederive the formulas for mean and variance directly from the mass function
$$f(k) = \frac{(k-1)!}{(k-r)!(r-1)!}p^r q^{k-r}, k = r, r+1, ...$$
.
3. A telemarketer makes a sequence of phone calls. After each phone call, he uses a chance device to decide whether to quit for the day (with probability a) or to make another call (with probability $1-a$). So the number of calls N is a random variable with a geometric distribution. Each call is successful with probability p. The random variable K is the number of successful calls. Calculate the expected number of calls that were made, given that exactly k calls were successful. That is, calculate the conditional expectation $\mathbb{E}(N|K = k)$.

4. In each of two consecutive games, a baseball player has five at-bats. This player has probability $p = 0.3$ of getting a hit in any given at-bat. You learn that the player has made a total of 4 hits in the two games. Conditioned on this, what is the mass function for the number of hits X in the first game? Compute the expectation and variance of X.

5. You roll 6 dice and get a random number X of sixes. You then roll these 6 dice again and get a random number Y of sixes. The random variable $Z = X + Y$ is the total number of sixes from all 12 die rolls.
   A is the event $X \geq 1$.
   B is the event $Z \geq 2$.
   Compute $\mathbb{P}(A), \mathbb{P}(B)$, and the conditional probabilities $\mathbb{P}(B|A)$,$\mathbb{P}(B^c|A)$,$\mathbb{P}(B|A^c)$, and $\mathbb{P}(B^c|A^c)$. Show that your answers agree with the solution to problem 3.8.5 in G&S.