MATHEMATICS 191, FALL 2003
MATHEMATICAL PROBABILITY
Assignment #1

Problems to be discussed in section: All problems are from Grimmett and Stirzaker, 1000 Exercises in Probability. The solutions are all in the book!

Add together the number of letters in your first and last name. If the sum is odd, prepare problems 1,3, and 5. If it is even, prepare 2, 4, and 6.

1. Section 1.2, problem 1.
2. Section 1.2, problem 4.
3. Section 1.3, problem 2.
4. Section 1.3, problem 4.
5. Section 1.8, problem 3.
6. Section 1.8, problem 21. Get a numerical answer for $p = \frac{1}{2}, r = 4, s = 3$.

Problems to be handed in on Thursday, Sept. 25:

Some of these problems rely on concepts from sections 1.4 and 1.5 of G&S.

1. You have to deliver crucial supplies using airplanes with very unreliable engines. Each engine has a probability p of lasting for the entire flight, and engine failures are independent events. If half or more of the engines fail, the plane crashes. Your choice is between using two-engine planes, which crash if either engine fails, or 4-engine planes, which crash if two or more engines fail.

   (a) What is the probability that exactly three engines on a four-engine plane will survive?

   (b) Determine for what value of p the probability that a plane will not crash is the same for 2-engine and 4-engine planes.

   (c) For this value of p, would 6-engine planes be a better choice?

2. The Queen of Sheba has come to Jerusalem to find a prophet. There are two sorts of prophets: true prophets, who speak the truth nine times out of ten, and false prophets, who speak the truth half the time. Prophet agents are forbidden to reveal explicitly which of their prophets are true ones.

   The queen wants to be more than 90% certain that the prophet she selects is a true one. She hires a prophet agent who brings out three prophets: two true ones and a false one. "2 out of 3 – that's not good enough, is it?" the agent asks. "Sure it is," says the queen, "as long as I can ask one yes-no question." "Ask away," says the agent.

The queen asks prophet 2, "Is prophet 3 a true prophet?" On hearing the answer she makes her selection and heads home. How did she do it?

Hint: Event B is "answer is yes", B' is "answer is no." Events A1, A2, A3 are respectively "prophet 1 is a true prophet," etc.

The queen must be sure that for either answer, her selection (made after hearing the answer) satisfies $P(A|B)$ (or $P(A|B')$) $> .9$

3. In the admissions office at Monty Hall University there are four interviewers. Three of them, Fl, F2, and F3, are friendly, while the fourth, U, is unfriendly. Every morning the Dean of Admissions assigns them randomly to offices 1, 2, 3, and 4, with an equal probability for each possible assignment. A student arrives for an interview and is asked to select which office he wants to be interviewed in. He chooses office 1 and learns that the interviewer in there is busy for the next half hour. "While you are waiting," says the Dean to the student, "I would like you to meet one of our friendly interviewers. From offices 2, 3, and 4, I will choose the lowest-numbered friendly interviewer." He opens the door of office 2 and introduces the student to an interviewer.
This is Event B – the lowest-numbered available friendly interviewer was in office 2.
Event A is that office 1 contains the unfriendly interviewer.

   (a) Enumerate all the ways of assigning interviewers to offices that lead to Event B. Assign a probability to each, and show that the sum of these probabilities equals the probability of Event B.

   (b) Given that Event B has occurred, determine the conditional probability of event A.

   (c) What is the probability that the friendly interviewer to whom the student was introduced by the Dean was F1?

4. Poker novice Jane picks up her five cards and asks "What did you say the probability was for event A (no two cards of the same rank)?" Veteran Betty tells her. Jane then says "Well, event B (all four suits represented in the hand) has just occurred for me. Is that worth anything?" Betty says, "No, but given that, do you want to know the conditional probability for A, which I'm planning to use when I bet against you?"
Calculate $P(A)$, $P(B)$, $P(A \cap B)$, and $P(A|B)$. You'll want a calculator.

5. The year is 2202, and Earth can support its large population only by having billions of people on cruise ships. Marcia Weis, the bridge director for Megaprincess Lines, is preparing a duplicate bridge tournament for about 10 million players. In the opening deal, she arranges for the North-South pairs all to have the same hands, including 9 spades – all but the queeen, 4, 3, and 2. She then distributes the remaining 26 cards (4 spades

and 22 others) in every possible way between East and West. Thus every declarer, in trying to guess how the missing cards are divided between East and West, will know that each possible outcome will occur precisely once on a cruise somewhere.

In what follows, let N = 22!/ (13!)(11!) = 4522 and express all answers as a multiple of N.

(a) How many different East hands of 13 cards can Marcia prepare?

(b) In how many of these hands does East have all four of the missing spades?

(c) In how many of these hands does East have all of the missing spades except the queen?

(d) In how many of these hands does East have precisely three of the missing spades?

(e) In how many of these hands does East have precisely two of the missing spades? (As a check, add together this number, twice the answer to part b, and twice the answer to part d. This should agree with the answer to part a).

(f) Since Princess passengers have been taught the maxim "Eight ever, nine never" at the mandatory lifeboat drill, they will all play this deal the same way, by leading the ace and king of spades and hoping that East or West will be forced to play the queen. This approach will succeed if East and West each has two spades or if either East or West has only the queen while the other has the remaining three spades. In how many of the hands will the approach succeed? What is the probability of success?

(g) Suppose that you are North, you lead the ace of spades, and both East and West play their lowest spade, which is not the queen. What is the conditional probability that East and West now each has one of the missing spades? What is the conditional probability that East has both of the missing spades?