MATH 191
PRACTICE EXAM
APRIL 13, 2001

Note: this practice exam is somewhat longer than the actual hourly. If you're planning on trying it with a time constraint, allow an hour and a half.

1. A single 6-sided die is rolled repeatedly until the third time a 6 comes up. Let $\mathbf{X}$ be the number of rolls up to and including the third 6, and let $\mathbf{Y}$ be the number of 5's rolled in the process.
   (a) Find the expectation and variance of $\mathbf{X}$.
   (b) Find the expectation of $\mathbf{Y}$.

2. A game is played as follows. Two red and eight black balls are placed in an urn, and you get to pick a subset of 4 of the 10 balls at random. You pay \$1 to play the game, and the payoff is \$1 if you get one red ball and \$3 if you get both. (If you pick 4 black balls you get nothing.)
   (a) What is your expected gain or loss each time you play?
   (b) What is the likelihood of breaking even or better after 100 games?

3. A game is played as follows: first we roll an n-sided die with faces numbered $1, 2, \ldots, n$. If the die comes up k, we then flip k coins. Let $\mathbf{X}$ be the random variable given by the number of heads that come up in the coin flips. Calculate the expectation and variance of $\mathbf{X}$

4. A very rudimentary slot machine costs \$1 to play, and pays off \$2 with probability .45 (and pays nothing 55% of the time). A fancier machine also costs \$1 to play, but has a flashier payoff structure: it pays off \$5 with probability .1, \$20 with probability .01 and \$200 with probability .001 (so that it pays nothing 88.9% of the time). Let $\mathbf{X}$ and $\mathbf{Y}$ be the random variables given by the payoffs of the two machines.
   (a) Find the expectations $E(\mathbf{X})$ and $E(\mathbf{Y})$.
   (b) Find the variances $\mathrm{Var}(\mathbf{X})$ and $\mathrm{Var}(\mathbf{Y})$.
   (c) Suppose you play either machine $1,000$ times. What are your chances of breaking even with each machine?
   (d) In view of the fact that the expectations of the two machines are the same, how would you explain the difference in the answers to part (c)?