Math 191 Solution Set 10

March 10, 2001

VI.4 The probability p that a specific player has four aces is $\binom{48}{9}/\binom{52}{13}$ (choosing 9 cards out of 48 non-aces). The probability of this player not get four aces in x deals is $(1-p)^x$; we need x so that $(1-p)^x \leq 1/2$. This requires $x \leq \log(1/2)/\log(1-p) \approx 262.1$, so 263 deals are necessary.

The probability that *some* player has four aces is $4p$, so the probability of nobody getting four aces in x deals is $(1-4p)^x$. Solving for $(1-4p)^x \leq 1/2$ yields $x \leq \log(1/2)/\log(1-4p) \approx 65.3$, so 66 deals are necessary.

VI.5 Note that $P\{\text{exactly } i \text{ hits on target}\} = \binom{10}{i}\left(\frac{1}{5}\right)^i\left(\frac{4}{5}\right)^{10-i}$. Hence
$P\{\text{at least two hits}\} = 1 - P\{\text{no hits}\} - P\{\text{one hit}\} = 1-(4/5)^{10} - 10(4/5)^9 \approx 0.624$.

VI.8 First we calculate the probability that the birthdays fall in two specified months (say A and B) and at least once in A and in B. Each birthday has a $2/12 = 1/6$ chance of being in A or B, so the probability that all birthdays are in A and B is $(1/6)^6$. However, this includes the cases that all birthdays are in one of the two months, so we need to subtract off $2(1/12)^6$. Hence the probability of the birthdays falling in A and B and at least one in each is $(1/6)^6 - 2(1/12)^6$.

Now to solve the actual problem, we can just multiply $(1/6)^6 - 2(1/12)^6$ by the number of ways to choose two months. Notice that there is no overcounting. Hence the final answer is $\binom{12}{2}\left((1/6)^6 - 2(1/12)^6\right)$.