Math 191 Solution Set 8

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V.1 If no two dice show the same face, then the first die has 6 choices, the second has 5 (to not duplicate the first die), and the third has 4. So $P(\text{no duplication}) = \frac{6\cdot5\cdot4}{6^3}$. If no two dice show the same face and one is an ace, then we have 3 choices for which die shows an ace, 5 choices for the face of a second die (any non-ace), and 4 choices for the last die (to not duplicate the first die and not be an ace). So $P(\text{no duplication and one is an ace}) = \frac{3\cdot5\cdot4}{6^3}$. Therefore, the desired conditional probability is $\frac{(6\cdot5\cdot4)/6^3}{(3\cdot5\cdot4)/6^3} = \frac{1}{2}$.

V.2 Note that $P\{\text{exactly } i \text{ aces}\} = \binom{10}{i}\left(\frac{1}{6}\right)^i\left(\frac{5}{6}\right)^{10-i}$. So $P\{\text{at least one ace}\} = 1-P\{\text{no aces}\} = 1-(5/6)^{10}$ and $P(\text{at least two aces}) = 1-P\{\text{no aces}\}-P\{\text{one ace}\} = 1-(5/6)^{10}-10(1/6)(5/6)^9$. Therefore, $p = P\{\text{at least two aces} \mid \text{at least one ace}\} = P\{\text{at least two aces and at least one ace}\}/P\{\text{at least one ace}\} = P\{\text{at least two aces}\}/P\{\text{at least one ace}\} = \frac{1-(5/6)^{10}-10(1/6)(5/6)^9}{1-(5/6)^{10}}$.

V.3 Given that West has no ace, there are 35 non-aces and 4 aces left in the deck. By the hypergeometric distribution, $P\{\text{partner has no aces} \mid \text{West has no ace}\} = \binom{35}{13}/\binom{39}{13} \approx 0.182$ and $P\{\text{partner has one ace} \mid \text{West has no ace}\} = \binom{35}{12}\binom{4}{1}/\binom{39}{13} \approx 0.411$. Thus $P\{\text{partner has two or more aces} \mid \text{West has no ace}\} = 1-P\{\text{partner has 0 or 1 ace} \mid \text{West has no ace}\} = 1-\binom{35}{13}/\binom{39}{13}-\binom{35}{12}\binom{4}{1}/\binom{39}{13} \approx 1-0.182-0.411 = 0.407$.

V.6 First, $P\{\text{defective}\} = P\{\text{defective} \mid \text{made by } A\}P\{\text{made by } A\} + P\{\text{defective} \mid \text{made by } B\}P\{\text{made by } B\} + P\{\text{defective} \mid \text{made by } C\}P\{\text{made by } C\} = (.05)(.25)+(.04)(.35)+(.02)(.40) = .0345$. Thus $P\{\text{defective} \mid \text{made by } A\} = P\{\text{defective and made by } A\}/P\{\text{defective}\} = P\{\text{defective} \mid \text{made by } A\}P\{\text{made by } A\}/P\{\text{defective}\} = (.05)(.25)/.0345 = 125/345$. Similarly, $P\{\text{defective} \mid \text{made by } B\} = 140/345$ and $P\{\text{defective} \mid \text{made by } C\} = 80/345$.