MATHEMATICS 154, SPRING 2009
PROBABILITY THEORY
Proof list for quiz #2

Last revised: March 25, 2009

Two of these proofs, chosen by tossing two dice, will appear on the quiz on Monday, April 6.

1. Prove that, if the random variable $X$ has the Poisson distribution with parameter $\lambda$, the expectation $\mathbb{E}(X) = \lambda$ and the variance $var(X) = \lambda$.

2. Formulate and prove the theorem that shows that the Poisson distribution is a limiting case of the binomial distribution.

3. First prove the tail-sum formula for the expectation of a discrete random variable. Then prove that the expectation of a geometric random variable is $\frac{1}{p}$, where $p$ is the probability of "success."

4. You start with $k$ chips and play a fair game ($p = \frac{1}{2}$) until you either have 0 chips or $N$ chips. Prove that the expected number of plays is $D_k = k(N - k)$.

5. State and prove the ballot theorem.

6. Prove that for a random walk of $2n$ steps that starts in level 0, the number of "nonnegative" walks is equal to the number of "return to zero" walks.

7. State and prove the Arc Sine law.