MATHEMATICS 154, SPRING 2009
PROBABILITY THEORY
Proof list for quiz #1

Last revised: February 22, 2009

Two of these proofs, chosen by rolling a die, will appear on the quiz.

1. State the axioms for a probability function, and use them to prove the inclusion-exclusion rule for two events A and B. Then extend the rule to the case of three events A, B, and C.

2. Let $A_1, A_2, \cdots$ be an infinite sequence of events. Starting from the generalization of Boole's inequality to the case of a countable infinity of events, prove that if the series

$$\sum_{n=1}^{\infty} \mathbb{P}(A_n)$$

is convergent, then the probability of the event A that infinitely many of the A_n occur is zero.

3. Using the generalized inclusion-exclusion rule, state and prove Waring's theorem.

4. State and prove the formula that resolves Simpson's paradox: i.e. derive a formula for $\mathbb{P}(A|B)$ in the case where event A is conditioned on two events B and C that are not independent.

5. Stating from the axioms for a probability function and the definition of the distribution function $F_X(x)$ for random variable X, prove that $F_X(x)$ is nondecreasing and that

$$\mathbb{P}(X = x) = F_X(x) - \lim_{h \to 0} F_X(x - h),$$

where the limit is one-sided, for positive values of h only.

6. Let S_n be the sum of n independent random variables, each of which is equal to 1 with probability p and 0 with probability $q = 1 - p$.

Given the identity $e^x \leq e^{x^2} + x$,

prove that for $\epsilon > 0$,

$$\mathbb{P}(\frac{S_n}{n} \geq p + \epsilon) \leq e^{-\frac{1}{4}n\epsilon^2},$$