

Name: \_\_\_\_\_

MATHEMATICS 154, SPRING 2009  
PROBABILITY THEORY  
Practice Quiz # 2  
for quiz on April 6, 2009

1. Two boxers fight a 10-round fight, and after each round the referee awards one point to the winner of that round. Boxer 1 wins the fight, six points to four. If all sequences of rounds are equally likely, what is the probability that at some point after the start of the fight the score was tied?
  - (a) 0.4
  - (b) 0.5
  - (c) 0.6
  - (d) 0.7
  - (e) 0.8
  
2. On Flight 154, six passengers have each requested a different special meal. The incompetent flight attendant distributes the six special meals on the flight randomly to the six passengers who ordered them. What is the probability that precisely four of the six passengers receive the meal that they requested?
  - (a)  $\frac{1}{24}$
  - (b)  $\frac{1}{30}$
  - (c)  $\frac{1}{48}$
  - (d)  $\frac{1}{60}$
  - (e) Approximately  $\frac{1}{2e}$

3. (a) Evaluate the sum

$$S = \sum_{k=0}^{\infty} \frac{k(k+2)!}{k!2!} x^k$$

- (b) A young MBA, after looking at dismal employment prospects, decides to embark on a career of white-collar crime. He operates in a state where after three convictions he will go to prison, but his lawyer assures him that his probability of success (avoiding conviction) for each crime is  $p = \frac{3}{4}$ . Write down (with a good, clear justification) the mass function  $f(k)$  for the probability that he will commit precisely  $k$  successful crimes before going to prison, and show that the probabilities sum to 1.
- (c) Calculate the expected number of successful crimes the MBA will commit before going to prison.

4. (5 points, 3/2) The wooden puppet Pinocchio was carved by Geppetto with a nose of length 1 inch, but his nose has grown whenever he told a lie and is now 3 inches long. If the nose grows to 4 inches, Pinocchio will become firewood, but if it shrinks back to 1 inch he will become a real boy.

Pinocchio looks in Geppetto's email inbox and finds lots of emails with titles like

Shr!nk your n@se by centimeters today!

Size d@es matter! Get back the n@se your maker g@ve you.

He replies to a sequence of these. Each reply has probability  $\frac{3}{4}$  of causing his nose to shorten by 1 inch and  $\frac{1}{4}$  of causing his nose to grow by 1 inch. Denote by  $f_k$  the probability that Pinocchio, starting with a nose  $k$  inches in length, will become a real boy.

- (a) Set up a recurrence for  $f_k$  (for  $k > 1$ ) and find two independent solutions to it.
- (b) Given that  $f_1 = 1$  and that  $f_4 = 0$ , determine  $f_3$ .

5. (4 points, 2/2) A Thanksgiving hostess is slicing a circular pie of radius 1 for a family member who has asked for "a random-sized piece of pie." She generates  $X$ , a random variable with a uniform distribution in  $[0, 1]$ , and then cuts a wedge of pie with angle  $Y = \arcsin(X)$ . So  $Y$  can assume any value in  $[0, \pi/2]$ .

- (a) Find the distribution function  $F_Y(y)$  for  $Y$  and find a density function  $f_Y(y)$  for  $Y$ . It is sufficient to give formulas that are valid for  $0 \leq y \leq \pi/2$ .
- (b) Calculate the expectation of  $Y$ . (There is more than one way to do this.)