Name: _____

MATHEMATICS 154, SPRING 2009
PROBABILITY THEORY
Practice Quiz # 1
February 2009

This year's quiz will have two proofs chosen at random from the list of six that was distributed. This would take the place of the last question.

1. In bridge, a player is dealt a hand of 13 cards chosen at random from a deck with 13 cards in each of 4 suits. Let $\mathbb{P}_{5431}$ be the probability that the longest suit in the hand has 5 cards and the others have 4, 3, and 1 cards. Let $\mathbb{P}_{4441}$ be the probability that the shortest suit in the hand has 1 card and the others all have 4 cards. The ratio $\frac{\mathbb{P}_{5431}}{\mathbb{P}_{4441}} =$

   (a) 0.72
   (b) $\frac{15}{16}$
   (c) $\frac{16}{15}$
   (d) 1.44
   (e) 4.32

2. The warning label on a drug reads: "Known side effects of this medication are drowsiness and headache. At least one of these occurs 50% of the time. Drowsiness occurs 20% of the time, and both side effects occur 15% of the time."

   What is the probability that if you take this drug you will develop a headache?

   (a) 0.15

   (b) 0.25

   (c) 0.3

   (d) 0.35

   (e) 0.45

3. A diligent but incompetent student has agreed to take an infinite sequence of mathematics tests. The event A_n is that he fails the nth test. The event A, which will lead to a Federal takeover of the student's school, is that he fails infinitely often.

   $\mathbb{P}(A) =$

   (a) $$\bigcap_{m=0}^{\infty} \bigcup_{i=m}^{\infty} A_i$$

   (b) $$\bigcup_{m=0}^{\infty} \bigcap_{i=m}^{\infty} A_i$$

   (c) $$\bigcap_{m=0}^{\infty} \bigcap_{i=m}^{\infty} A_i$$

   (d) $$\bigcup_{m=0}^{\infty} \bigcup_{i=m}^{\infty} A_i$$

   (e) $$\bigcap_{m=0}^{\infty} \bigcup_{i=0}^{m} A_i$$

4. On a reality TV show, participants have voted to execute one of their number by shooting, and you are chosen to do the job. There are four identical-looking rifles. Three have live ammunition; one contains a blank cartridge. You choose one at random and prepare to pull the trigger. Surprise: Monty Hall, who knows which rifle has the blank cartridge, appears. He chooses one at random from among the other rifles with live ammunition, fires it into the air, and invites you to swap your rifle for one of the two remaining guns. If you accept his offer, the probability of ending up with the blank cartridge is

   (a) $\frac{1}{4}$
   (b) $\frac{1}{3}$
   (c) $\frac{3}{8}$
   (d) $\frac{1}{2}$
   (e) $\frac{2}{3}$

5. (3 points) Each resident of northern Alaska is to be allocated a square patch of tundra on which to drill for oil. The side of the square is determined by a random variable X that is equally likely to have any value between 1 and 4. However, the result is then rounded up if necessary so that the area of the square is at least 4.

   The random variable Y is the area of the square. Its distribution function is $F_Y(y)$. Determine $F_Y(4)$, $F_Y(9)$, and $F_Y(16)$, then sketch a graph of $F_Y(y)$.

6. (5 points, 1/1/1/2) Every day, a Web site posts a "four-letter word of the day," and students have become fond of betting on its properties. A clever mathematics student hacks the Web site and determines the following:

   For "event A" (the word begins with a consonant), $\mathbb{P}(A) = 0.8$.

   For "event B" (the word ends with a vowel), $\mathbb{P}(B) = 0.3$.

   For "event C" (the word begins and ends with a consonant), $\mathbb{P}(C) = 0.6$.

   (a) Using set-theoretic notation, express event C in terms of events A and B.

   (b) Are events A and B independent? Justify your answer.

   (c) Event D is that the word has a consonant at the beginning, at the end, or both at the beginning and at the end. Express this event in terms of A and B, and calculate its probability.

   (d) If you know that today's word ends in a vowel, what is the conditional probability that it also begins with a vowel?

7. (4 points) Given that

$$\mathbb{P}(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} \mathbb{P}(A_i),$$

   prove that

$$\mathbb{P}(\bigcup_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} \mathbb{P}(A_i) - \sum_{i<j} \mathbb{P}(A_i \cap A_j).$$

   Do the case $n = 2$ directly, then do the general case by induction. You may assume the inclusion-exclusion principle for two events.