Math153: Problem set 3. Due 11 November 2004.

Problem 1: Hawk, Dove and Mixed

In the Hawk Dove game there exists a mixed strategy which is evolutionarily stable. This strategy is given by playing Hawk with probability b/c and playing Dove with probability $1-(b/c)$. Consider the interaction between three strategies: (i) pure Hawk, (ii) pure Dove, and (iii) the above mixed strategy.

Confirm that the payoff matrix for these three strategies is given by:

$$A = \begin{array}{c} \\ \text{Hawk} \\ \text{Dove} \\ \text{Mixed} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Hawk} & \text{Dove} & \text{Mixed} \end{array} \\ \begin{pmatrix} \frac{b-c}{2} & b & \frac{b}{2}(1-\frac{b}{c}) \\ 0 & \frac{b}{2} & \frac{b}{2}(1-\frac{b}{c}) \\ \frac{b}{2}(\frac{b}{c}-1) & \frac{b}{2}(1+\frac{b}{c}) & \frac{b}{2}(1-\frac{b}{c}) \end{pmatrix} \end{array}$$

Analyze the evolutionary dynamics given by the replicator equation

$$\dot{x}_i = x_i(\sum_{j=1}^{3} a_{ij}x_j - \phi) \qquad i = 1..3$$

Choose parameter values for b and c. Plot trajectories for different initial conditions on the simplex S_3. Indicate all fixed points.

What happens if you add mutation?

Problem 2: ALLD, ALLC and TFT

Consider a repeated Prisoner's Dilemma. There is a probability w to play another round. The average number of rounds is given by $\bar{m} = 1/(1-w)$. Calculate the payoff matrix for ALLD, ALLC and TFT. Study the replicator dynamics on the simplex S_3.

Choose parameter values. Plot trajectories for different initial conditions and indicate all fixed points. What happens in the limit of the infinitely repeated Prisoner's Dilemma, $w \to 1$?

What happens if you introduce a complexity cost for TFT by reducing its payoff values by a small constant δ ?

What happens if you add mutation? (With and without the complexity cost.)