Math153: Problem set 4. Take home final. Consult everything but work alone. Due 10 January 2004.

Problem 1: Language evolution

Consider the language evolution equation

$$\dot{x}_i = \sum_{j=1}^{n} x_j f_j(\vec{x}) Q_{ji} - \phi x_i \qquad i = 1, .., n$$

with $f_j(\vec{x}) = \sum_{i=1}^{n} F_{ji} x_i$ and $\phi = \sum_i x_i f_i$.

Explain the assumptions of this equation. Which process is described here?

Let $Q_{ii} = q$ for all $i = 1..n$. Let $Q_{ij} = (1-q)/(n-1)$ for all $i \neq j$.

Let $F_{ii} = 1$ for all $i = 1..n$. Let $F_{ij} = a$ for all $i \neq j$.

Discuss the bifurcation diagram of this equation. Proof existence and stability of fixed points. What is the minimum learning accuracy that is needed for grammatical coherence?

Problem 2: Evolutionary graph theory

Consider an evolutionary graph, G, with n vertices. At time 0, all vertices are occupied by individuals of type A. In each reproductive event, A can mutate into B with probability u. B does not mutate into A. B has relative fitness r. This stochastic process has only one absorbing state: all-B. We are interested in finding a graph G that minimizes the time to reach this absorbing state.

Write a stochastic simulation of this process. You will need a random number generator.

Let $n = 10$, $u = 10^{-4}$ and $r = 1$. Try different graphs. Do at least 1000 runs for each graph. Compute the average time until absorption into all-B. Which graph is fastest? Why?

Keep the same n and u and find the fastest graph for $r = 2$.

Find fast graphs for $n = 100$; (try $r = 1$ and $r = 1.1$)

Supply a summary of your results and a file with a listing of your computer program. Discuss your finding.

Problem 3: Evolution of virulence

Let x and y denote uninfected and infected hosts. Deterministic infection dynamics are given by

$$\begin{aligned} \dot{x} &= \lambda - dx - \beta xy \\ \dot{y} &= \beta xy - ay \end{aligned}$$

Discuss all equilibrium points of this differential equation. Give conditions for their existence and stability. What is the basic reproductive ratio of an infection? Why is this an important quantity?

Consider competition of two different parasites in the same population of hosts. Infection dynamics are given by

$$\begin{aligned} \dot{x} &= \lambda - dx - \beta_1 x y_1 - \beta_2 x y_2 \\ \dot{y}_1 &= \beta_1 x y_1 - a_1 y_1 \\ \dot{y}_2 &= \beta_2 x y_2 - a_2 y_2 \end{aligned}$$

Proof that only one parasite strain can survive. This is called competitive exclusion. Which strain will survive? Discuss some basic ideas for the evolution of virulence. Which modifications of the equation would lead to coexistence between the two strains.