



**Last Revised:** June 29, 2005

### Course Meeting Place

- The course meets on Tuesdays and Thursdays at 3:30-6:00 P.M. in Science Center 109.
- Problem sections meet on Mondays and Wednesdays at 7:00-8:30 P.M. in Science Center 112.

The course will be run in a seminar style, with most of the topics presented by students in the class. This means that your classmates will be counting on you to prepare carefully and that you will gain lots of experience in presenting proofs at the blackboard.

### Course Description

An introduction to finite groups, finite fields, finite geometry, discrete probability, and graph theory. A unifying theme of the course is the symmetry group of the regular icosahedron, whose elements can be realized as permutations, as linear transformations of vector spaces over finite fields, as collineations of a finite plane, or as vertices of a graph. Taught in a seminar format, students gain experience in presenting proofs at the blackboard. Students have the opportunity to implement some of the key mathematical ideas of the course in C++, but computer work is optional. Prerequisites: Elementary knowledge of vectors, 2x2 matrices, and determinants. [MATH S-21b](#), taken previously or concurrently, would be ideal. Calculus is not required. Placement test recommended.

### Course Website

<http://www.courses.fas.harvard.edu/~sum31731/>

### Instructor

Thomas W. Judson  
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### Office Hours

Tuesday and Thursday at 2:00-3:00 P.M.

### Course Assistant

Michael Polansky  
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### Textbook

- Norman L. Biggs. Discrete Mathematics, second edition. Oxford University Press, New York, 2002.
- You will also need to obtain the course pack of materials. Details on how to obtain the course pack will be announced in class.

**Course Goals and Prerequisites**

This course will introduce you to a variety of topics in higher mathematics that are “discrete” in the sense that they are not dependent on limits and approximation. Ideas from geometry, group theory, rings and fields, graph theory, linear algebra, combinatorics, and probability will be studied, and surprising connections will emerge. The unifying theme of the course is rotational symmetries of the so-called Platonic solids, especially the regular icosahedron.

You are expected to have a background in linear algebra (like Math 21b or a course that you took elsewhere) and an interest in theoretical mathematics. Previous experience with proofs is not necessary. One of the aims of the course is to introduce you to the techniques of proof in higher mathematics.

Because the subject matter of the course is discrete, calculus is irrelevant.

**Grading and Exams**

Your course grade will be determined by your homework, class presentations, quizzes, exploratory homework and/or programming assignments, and a final exam.

- Required Homework - 20% (We will drop your two lowest homework assignments.)
- Class Presentations - 10%
- Exploratory Homework and/or Programming Assignments - 10%
- Two Best Quizzes - 20%
- Third Quiz - 5%
- Take-Home Final Exam - 35%

Range of Numerical Values	Corresponding Letter Grade
100 - 90	A
89 - 80	B
79 - 65	C
64 - 50	D
49 - 0	E

Your grade will be determined by the table above. When we calculate your final grade at the end of the course, we will calculate a score on a 0-100 point scale using the scores that you have obtained during the course, and using the grade breakdown given below. Your course grade will then be obtained using this table. In the event of a fractional score, we will always round up to the nearest integer. We may modify these letter grades with a “+” or a “-” if we believe that your performance in the

course warrants this. Make-up exams will be administered only if a documented serious illness or personal tragedy prevents a person from taking an exam at the scheduled time.

### Homework

There is no question that the best way to learn math is by doing math, and homework exercises are an essential part of any math course. If you just go to a math class and watch the teacher work problems, but do not actually try doing any problems on your own, then there is very little chance you will really learn the subject. It is also very unlikely that you will do well on exams without working through homework problems ahead of time. While doing homework, do not just write down answers. Think about the problems posed, your strategies, the meaning of your computations, and the answers you get. The main point is not to come up with specific answers to the specific problems you are working on, but to develop an understanding of what you are doing so that you can apply your reasoning to a wide range of similar situations. It is very unlikely that later on in life you will see exactly the same math problems you are working on now, so learn the material in such a way that you are prepared to use your general knowledge of mathematics in the future, not just how to apply particular formulas for very specific problems.

We encourage you to form study groups with other students in the class so that you can discuss your work with each other; however, all work submitted must be written up individually. Make sure that even if you do work in groups, that you come away with the ability to explain everything you end up writing up in your homework.

There will be usually be one or two problem sets each week. Assignments will be graded by your course assistant and will typically be returned to you at the following class meeting. Since we will drop your two lowest homework grades, please do not try to harass your course assistant into accepting a late homework assignment.

There are two options for the second homework component of the grade. This work may be completed any time before the last day of class (Thursday, August 11, 2005) though it is recommended that you begin long before then. The first option is a set of exploratory problems (approximately 20 problems) that will engage your creativity, consisting of some more difficult proofs and some open-ended questions. The second option is a set of four programming assignments for those more interested in computer science. It is possible to mix and match from among the exploratory problems and computer assignments.

### Approximate Day-by-Day Syllabus

June 28, 2005	Counting, Symmetries and Platonic Solids
June 30, 2005	Permutations and Groups
July 5, 2005	Congruence Arithmetic
July 7, 2005	Subgroups and Quotient Groups, Quiz #1
July 12, 2005	Rings
July 14, 2005	Fields
July 19, 2005	Finite Affine Geometry
July 21, 2005	Finite Affine Geometry, Quiz #2
July 26, 2005	Linear Algebra over Finite Fields

July 28, 2005	Group Isomorphisms
August 2, 2005	Set Theory and Probability
August 4, 2005	Probability, Countability and Uncountability, Quiz #3
August 9, 2005	Graph Theory, Trees, and Spanning Trees
August 11, 2005	Generators, Graphs, and Groups
August 15, 2005	Take-Home Final Exam Due

For a more accurate day-by-day syllabus and the corresponding readings, see the course web page.