MATHEMATICS 152, FALL 2004
METHODS OF DISCRETE MATHEMATICS
Homework Problems Relevant to the Final Exam
Due: Thursday, January 13, 2004

Required Problems

Note: in calculating homework scores, we will implement a "throw out your worst set" policy by coveting your worst grade (in terms of missing points) to a perfect score before totaling and averaging. If you have done well on all other problem sets, you may lose little by not handing this in. However, you will need to know how to do the problems in order to be well prepared for the final exam!

1. Biggs, p. 179, exercise 15.1.1.
2. Biggs, p. 191, exercise 15.8.3.
3. Biggs, p. 185, exercise 15.4.4.
4. Biggs, p. 187, exercise 15.5.1.
5. Biggs, p. 187, exercise 15.5.3.
6. Biggs, p. 202, exercise 16.3.4.
7. Imagine that United Airlines, having already cut back its service on Saturdays to the graph shown as "united.aaa" in the notes, decides to go further and cut back its service to a minimal spanning tree.
    (a) Show the order in which routes (edges) will be added to form the minimum spanning tree if you start in Washington and use Prim's algorithm (the one described on Biggs section 16.3)
    (b) Show the order in which routes (edges) will be added to form the minimum spanning tree if you use Kruskal's algorithm (described only in the notes, where during the construction of the spanning tree the selected edges may fail to form a single tree.)

Exploratory Problems

Here is a very large set, so that you can have plenty of choice. Most of the problems cover material discussed at the last class, on January 13. You may hand in solutions to these problems at the final exam. If you are sure that you already have the maximum of 50 points from exploratory problems and programming projects, please do not hand them in! Do them, though, especially the final three, since they provide a nice review of symmetry groups for the final exam.

1. Biggs, p. 191, exercise 15.8.13.

2. Biggs, p. 191, exercise 15.8.14.

3. Grossman and Magnus, page 68, Exercise 15.

4. Grossman and Magnus, page 68, Exercises 16 and 17.

5. Grossman and Magnus, page 119, Exercise 51.

   For the remaining problems, use groups.exe to multiply permutations.

6. Draw a depth-first spanning tree for the graph of the tetrahedral group A_4 (Grossman and Magnus, figure 10.12), starting from the identity element I. Using as generators $r = (134)$ and $f = (13)(24)$, label each vertex of the tree with a permutation.

7. Suppose that the generators for A_5 are taken as $r = (12345)$ and $f = (12)(34)$.

   (a) Show that $(rf)^3 = 1$, and find a different element of order 5 for which rf does not have order 3.

   (b) The graph of the icosahedral group (Grossman and Magnus, figure 16.2) shows a central pentagon with five vertices of order 2. Express these group elements as words in r and f (without using r^{-1}). Express them as permutations by using the specific r and f given above.

8. (a) Make a copy of the graph in the notes that represents the symmetry group of the cube, S_4.

   (b) Using as generators $r = (1432)$ and $f = (34)$, label each vertex of the graph with a permutation. For the sake of uniformity, take the identity to be the top left vertex of the outer square and put (1432) at the bottom left vertex of the outer square. This will lead to a different labeling than is given in the notes.

   (c) Find a closed path in the graph that illustrates the relation $(rf)^3 = 1$.

   (d) Circle the vertices of the graph that correspond to the elements of the subgroup A_4.