MATHEMATICS 152, FALL 2002
METHODS OF DISCRETE MATHEMATICS
Homework Assignment #11
Due: January 8, 2004

Required Problems

1. Biggs, p. 179, exercise 15.1.1.
2. Biggs, p. 181, exercise 15.8.3.
3. Biggs, p. 185, exercise 15.4.4.
4. Biggs, p. 187, exercise 15.5.1.
5. Biggs, p. 187, exercise 15.5.3.
6. Biggs, p. 191, exercise 15.8.13.
7. Use the algorithm presented in class to construct an Eulerian cycle for K_5. Construct the original cycle with six solid edges, then construct a Eulerian cycle, shown with dotted edges, for the small subgraph that remains. Explain how you combine the two cycles into a single Eulerian cycle for the entire graph.
8. Imagine that United Airlines, having already cut back its service on Saturdays to the graph shown as "united.aaa" in the notes, decides to go further and cut back its service to a minimal spanning tree.
   (a) Show the order in which routes (edges) will be added to form the minimum spanning tree if you start in Washington and use Prim's algorithm.
   (b) Show the order in which routes (edges) will be added to form the minimum spanning tree if you use Prim's "greedy" algorithm.
9. Biggs, p. 204, exercise 16.4.3.
10. Biggs, p. 206, exercise 16.5.2.
11. Biggs, p. 208, exercise 16.6.1.

Exploratory Problems

Because the difficulties with Linux in the Science Center have made it unexpectedly difficult to do the programming projects, here is a very large collection of exploratory problems on graph theory, so that you can have plenty of choice if you need to get all 40 points from exploratory problems. Most of the following problems cover material that will be not discussed in class until January 8. You may hand in solutions to these problems at the final exam. If you are sure that you already have the maximum of 40 points from exploratory problems and programming projects, please do not hand them in! Do them, though, especially the final three, since they provide a nice review of symmetry groups for the final exam.

1. Biggs, p. 191, exercise 15.8.3.
2. Biggs, p. 191, exercise 15.8.14.
3. Grossman and Magnus, page 68, Exercise 15.
4. Grossman and Magnus, page 68, Exercises 16 and 17.
5. Grossman and Magnus, page 119, Exercise 51.

   For the remaining problems, feel free to use groups.exe to do the computation of permutations.

6. Draw a depth-first spanning tree for the graph of the tetrahedral group A_4 (Grossman and Magnus, figure 10.12), starting from the identity element I. Using as generators $r = (134)$ and $f = (13)(24)$, label each vertex of the tree with a permutation.
7. Suppose that the generators for A_5 are taken as $r = (12345)$ and $f = (12)(34)$.
   (a) Show that $(rf)^3 = 1$, and find a different element of order 5 for which rf does not have order 3.
   (b) The graph of the icosahedral group (Grossman and Magnus, figure 16.2) shows a central pentagon with five vertices of order 2. Express these group elements as words in r and f (without using r^{-1}). Express them as permutations by using the specific r and f given above.

8. (a) Draw a graph to represent the symmetry group of the cube, S_4. The easiest way to do this is first to construct a large outer square and a small inner square. Then join each vertex of the outer square to the nearest vertex of the inner square using two dotted edges, between which is inserted an intermediate square with two of diagonally-opposite vertices connected to the dotted edges. Finally, join up the remaining vertices of the intermediate squares with four dotted edges. For the outer square, edges go counterclockwise. For the inner square and intermediate squares, they go clockwise. The 12 edges that join different squares (corresponding to the order 2 generator) should all be dotted.

   (b) Using as generators $r = (1423)$ and $f = (34)$, label each vertex of the graph with a permutation. For the sake of uniformity, take the identity to be the top left vertex of the outer square and put (1423) at the bottom left vertex of the outer square.

   (c) Find a closed path in the graph that illustrates the relation $(rf)^3 = 1$.

   (d) Circle the vertices of the graph that correspond to the elements of the subgroup A_4.