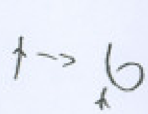
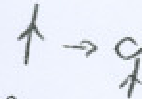
Problem 1 P 52, 1

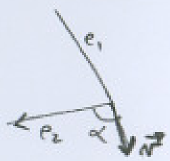
The 1-moves  and  change the writhe by ± 1 w/o affecting the link. We apply them to any given diagram to get $w(L) = 0$.

Problem 2 [Tim]

If K_1 & K_2 are the two link components, color the projection of K_1 using the chessboard coloring. Then at each crossing between K_1 & K_2, the strand in K_2 goes from a black square to a white one or viceversa. Since we must start & end in the same color when we travel once around K_2, we must have an even # of crossings between K_1 & K_2. Each of this crossings adds 1 mod 2 to the signed sum of crossings, which then turns out to be even. Hence the linking number is even, and we are done.

∎

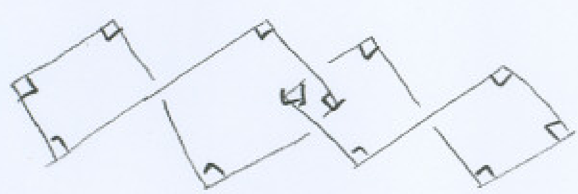
Problem #3



Assume we start travelling from $\vec{v}$ shown in the diagram to the right. $\vec{v}$ is contained in the plane determined by e_1 + e_2 so, as it travels from e_1 to e_2, its unitary representative centered at the origin traces a great circle arc on S^2. As $\vec{v}$ travels around the knot and returns to its startpoint, its representative traces a closed curve on S^2. The winding # of the knot is then $\frac{1}{2\pi}$ times the angle change of the representative vector. But since we traced a closed curve of great circle arcs, this # must be an integer.

Problem 4

For links, the answer is yes!



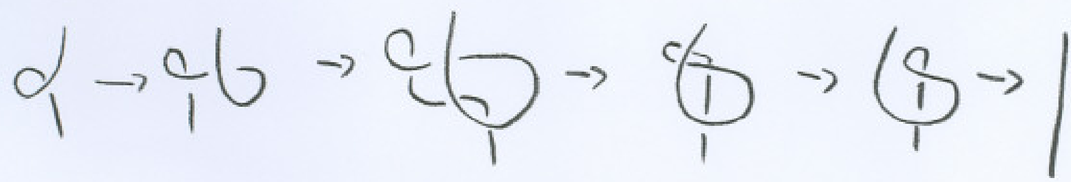
writhe = 0
winding # = 0

for knots, the answer is no!

In general, the difference (winding # - writhe) is 1 (mod 2) for all knots:

Note that we can unknot a knot by changing crossings in any of its diagrams, and each switch changes writhe by 2. Furthermore, 1-moves change both writhe + w.n. by 1 and R_2, R_3 leave both unaffected. We can use these moves to transform any knot diagram into a planar S^1, which has w.n=1 and writhe 0, this proves the claim.

Problem 5

- 
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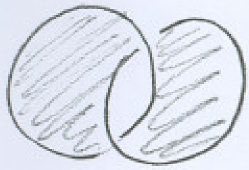
$\blacksquare$

Problem 6

Hopf



→ black surface :

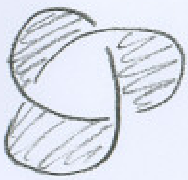
orientable!

→ white surface :

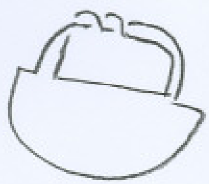
orientable!

[black here just to illustrate... <u>not</u> the same black as before]

Trefoil



→ black



un orientable! (odd # of twists)

→ white : orientable

remove ∞ from bottom disk

~

0 mod 2 0 mod 2

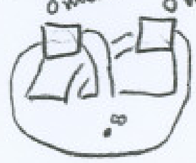
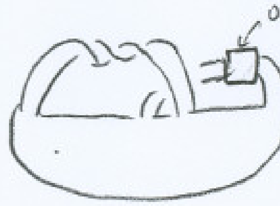


Figure 8

→ black : unorientable

→

0 mod 2



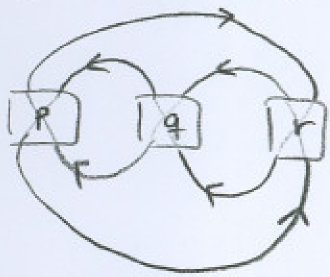
→ white : orientable

seifert surface!

■

Problem 7

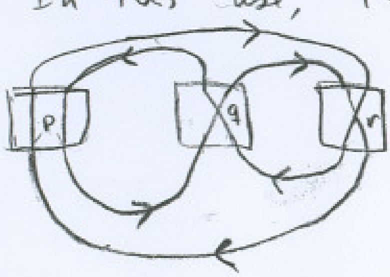
If p, q, r are all odd, the knot may be oriented as



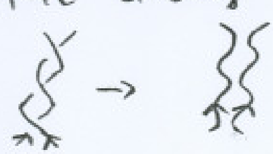
so the seifert algorithm produces $(|p|-1)+(|q|-1)+(|r|-1)+2$ disks. Since there are $p+q+r$ crossings and $\mu(D)=1$,

the genus is $\frac{1}{2}\left(1-\left((|p|-1)+(|q|-1)+(|r|-1)+2\right)+(|p|+|q|+|r|+1-1)\right) = 1$

From pset 1, the case $(p,q,r)=(0,1,1)$ mod 2 suffices to show what happens when only one of the (p,q,r) triplet is even. In this case, the orientation is given by the diagram



Then the $q + r$ produce no Seifert circles;



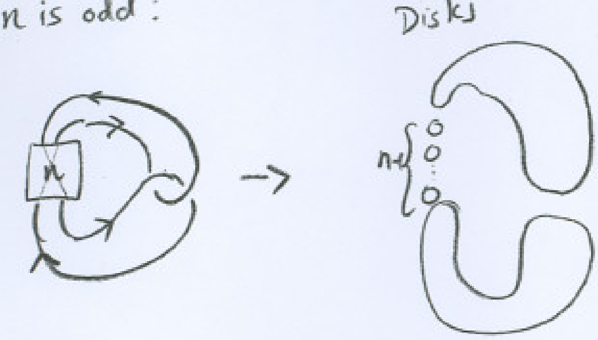
and hence the genus is

$$\frac{1}{2}\left(1-|p|-1+|p|+|q|+|r|\right) = \frac{|q|+|r|}{2}$$

$\square$

Problem 8

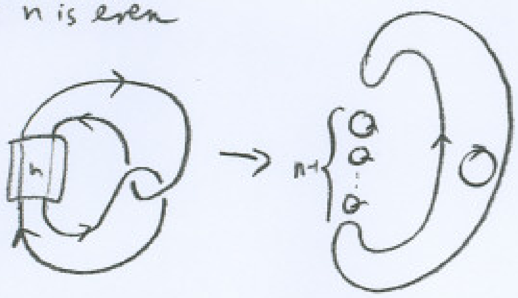
• If n is odd:



we have $n+2$ crossings and $(n-1)+2 = n+1$ disks, so the genus is

$$\frac{1}{2}\left(1-(n+1)+(n+2)\right) = 1$$

• If n is even



We have $n+2$ crossings and $n+1$ disks, so by the same computations above, the genus is 1.

Problem 8

[Diagrams by Eleanor]