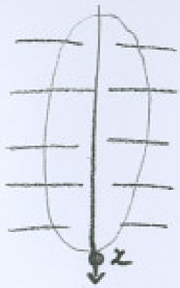
Problem #5

Consider the 1-bridge presentation:

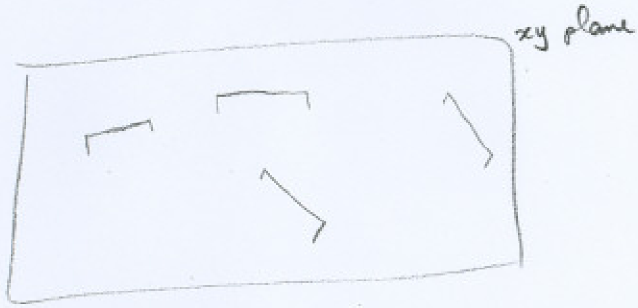


Pick a point x away from the crossings, on the bridge. We construct an ascending diagram by following the direction away from the circle shown.

□

Problem #6

Consider an embedding of the n bridge knot where only the bridges rise from the xy-plane and the bridges are three sides of a rectangle which rises up to $z=1$.

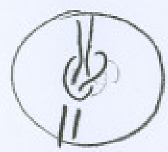


A box with base side at $z=1/2$ separates the knot into 2 trivial tangles: n disjoint disks at the top and an unknotted, unlinked curve at the xy plane. □

Problem #3

Consider the embeddins of the unknot with the sphere shown:

one of the tangles is

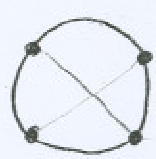


which contains the trefoil + hence it is non-trivial.

Problem #5

Rational links can be decomposed into 2-trivial 2-tangles.

Hence, they are given by joining 4 points on a sphere

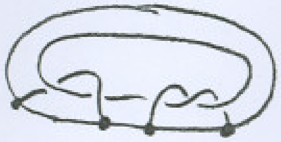


This yields at most 2 components (3 components or more would require 6 or more crossings).

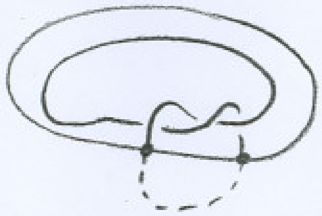
Problem #2 [partial]

5.2

The top tangle is



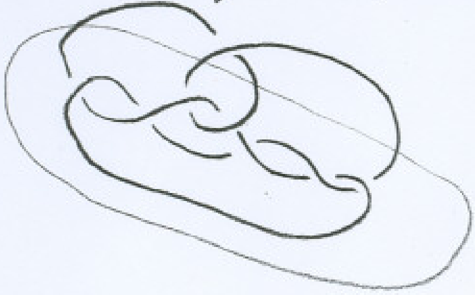
this is non trivial since we obtain a tangle decomp. of the trefoil by adding a ~~tan~~ trivial tangle:



Since the trefoil is knotted and the dotted lines are a trivial tangle, then the original tangle is non-trivial.

the other part is harder and we're not doing it.

I think this might work, though:



#1 (Kristen's picks)

 is just a trefoil.

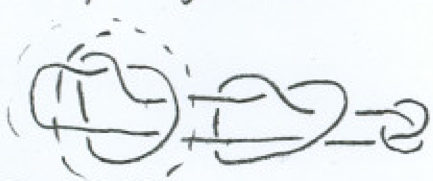
Consider its tangle decomposition given by the sphere shown.

The tangles are

 and 

which are both non-trivial.

Consider the trefoil again, this time as



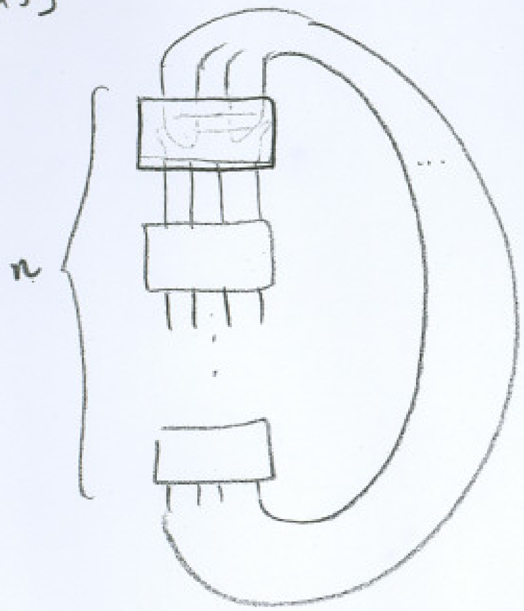
with the decomposing sphere shown.

This new decomposition into non-trivial tangles is ≠ from the one above and again non-trivial.

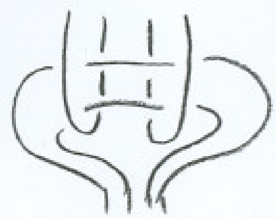
∎

c) A factorizing sphere must intersect one of the components. Call it k. Note that k is trivial because the link is brunnian. If any components of L were outside of the sphere, then the inside of the sphere has a trivial link again because the link is brunnian. So only k leaves the sphere. Since it is unknotted, our original factorization is invalid. ∎

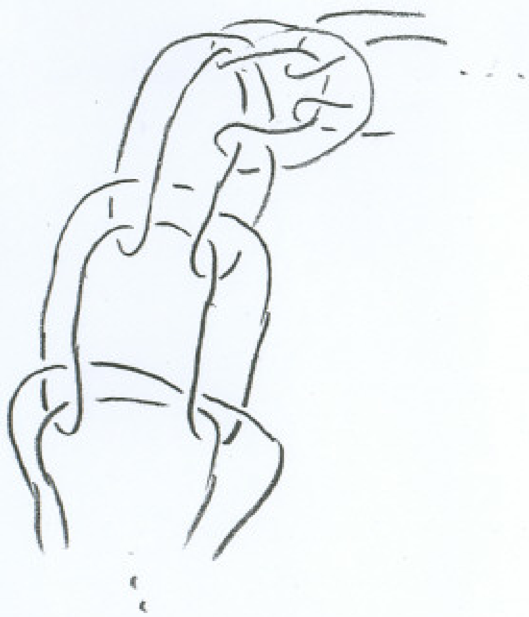
[Tim's]



each box:



[Kristen, Elizabeth, Alison et al] drew this as:



Every knot is its own companion by taking a trivial pattern.

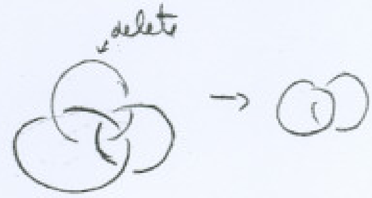
If A is a companion of B and B one of C, place C in a tubular nbhd of B and B inside a tubular nbhd of A. Then C lies in a tubular nbhd of A. Essential-ness is not altered by homeomorphisms, so we are done. ∎

Problem #9

Factors of a knot are companions by swallow-follow. So a knot with no proper companions must be prime. ∎

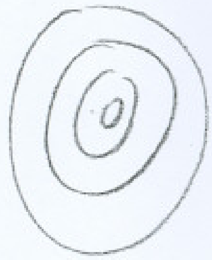
Problem #10

a) this is trivial:



b) we have 2 constructions.. They are actually the same, but they're both cool:

nested



and now we just plug in:

$$2g(F) = 1 - s(D) + c(D)$$

$$= 1 - (q) + (p(q-1))$$

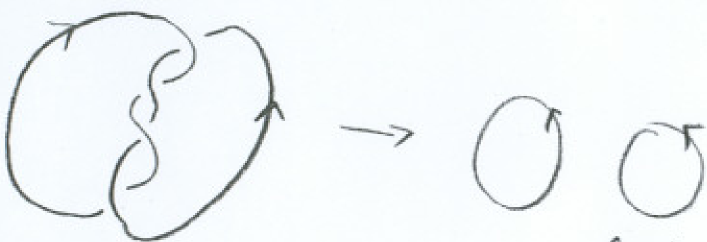
$$= 1 + pq - p - q = (p-1)(q-1)$$

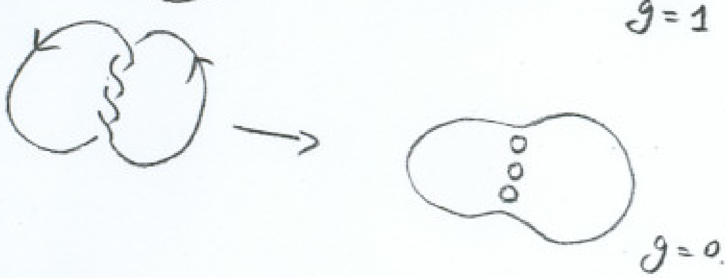
so $g(F) = \frac{1}{2}(p-1)(q-1)$

∎

Problem 6

Consider





∎

Problem #4

We know that as knots (link)

$$C(-n,-2) = P(-n,-1,-1) = P(-n+1,3,-1)$$

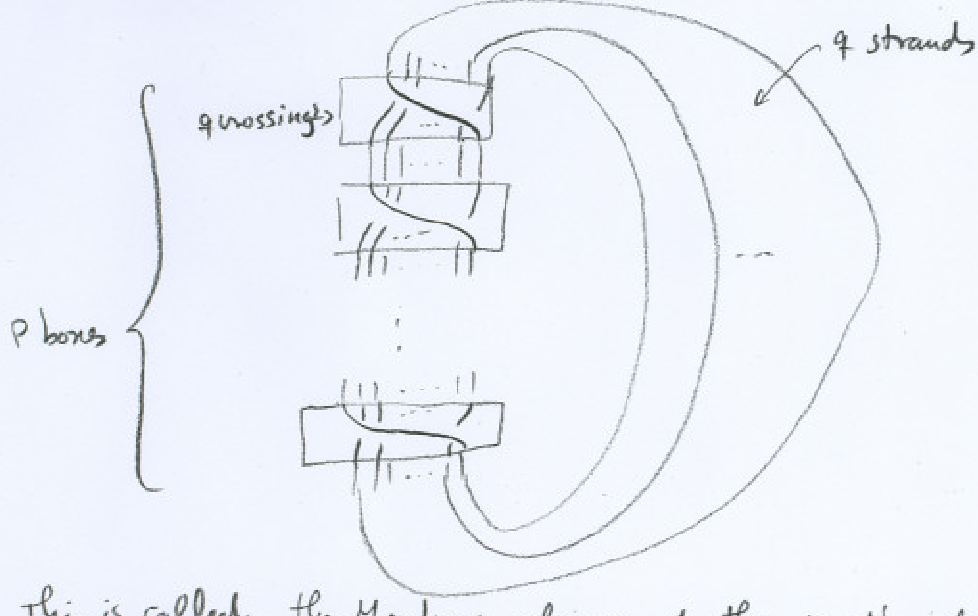
If n is odd — If n is even

Select the diagrams as we indicate above. By HW2, these diagrams give a projection surface of genus 1.

Problem #5

[Some people had beautiful pictures, but this is the clearest one, I think.] It's also how I always draw torus knots now]

A p,q torus knot has a diagram



This is called the Markov closure of the knot's preferred braid presentation

Represent each disk as a r-gon, where r is twice the number of bands that are attached to it and bands are attached to it at even-labelled edges (yes-no-yes-no...)
Divide each band into 2 Δ's diagonally. Each disk has contribute 1 to χ. In each band, consider 2 Δ's and one edge, since 2 edges and all 4 vertices are part of a disk. Then each band contributes -1 to χ. Then the Euler characteristic is $\chi = n-m$.

Then by definition,

$$\begin{aligned} 2g &= 2-\chi(F) - |\partial F| \\ &= 2-n+m-|\partial F| \end{aligned}$$

$\boxed{}$

#1

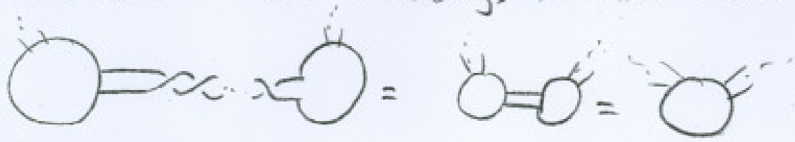
γ_1 is not essential or separating in either surface

γ_2 is separating but not essential in the möbius band
essential but not separating in the annulus

γ_3 is separating in both cases
not essential in either one

γ_4 is not separating in the Möbius band, but does satisfy the 3 other properties.

2) If we have 2 disks and a band connecting, the claim is trivial.



Assume we have n+1 disks, and that the statement is true for any n disks. Isotope the surface so one of the disks lies inside a sphere S and all other disks are disjoint from it.* Let f be a homeomorphism of the outside surface to a disk with bands attached. We may assume f preserves the intersection of the surface with the sphere. Alternatively, apply f and glue the images of the intersections with the sphere to the original intersections. Then the original surface is homeomorphic to one with only one disk, and we are done. ∎

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*only bands intersect f, and they do so transversally