

Homework 6

Math 139

DUE: Thursday March 22, 2007.

Reference: Knots and Links Chapter 6.

0. Don't forget to submit your 1 page summary of your project in class on Wednesday March 21, 2007.
1. Read Theorem 6.1.2 and its proof (page 131 in Knots and Links). To complete the proof you need show that the coefficients constructed for the 1-cycle z are unique.
2. In this question you will practice finding the determinant and signature of a knot.
 - (a) Choose a knot with crossing number ≥ 5 and find a projection surface F for it.
 - (b) Find the Seifert matrix. Please only include a picture showing the surface with the basis elements for $H_1(F)$, the matrix and maybe one or two pictures showing how you found the linking number of one or two pairs of loops.
 - (c) Find the determinant and signature of the knot. Check your answer using Appendix B pages 294—296 in Knots and Links.
3. Read Lemma 6.7.6 and its proof (page 148 in Knots and Links). Complete the proof as indicated.
4. Construct an example to show that the determinant of a Seifert matrix is not a link invariant.
5. Look at the table in Figure 6.6 (page 146 in Knots and Links). The pattern of blank and filled in entries in the table make it look a bit like a chess board pattern.
 - (a) Show that when K is knot, $\text{sign}(\det(K)) = (-1)^{\sigma(K)/2}$.
 - (b) Hence deduce that $\sigma(K) = \det(K) - 1 \pmod{4}$. This verifies the chessboard pattern.
6. Ask Gerardo to talk about determinant of a satellite knot and how signature can help you find out a bit more the unknotting number of a knot.