

Homework 4

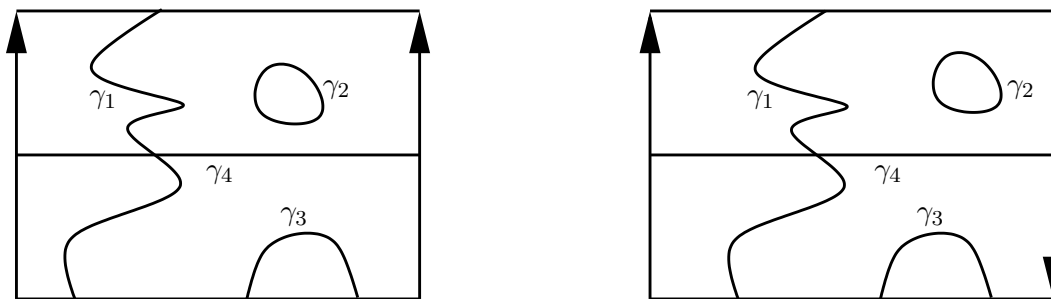
Math 139

DUE: Thursday March 8, 2007.

Reference: Knots and Links Chapters 2, 4 and 5.

0. Take a look at the project page. At some point before 9th March come and visit me and choose a project to read. We can try and find something that matches your interests reasonably closely.

1. The Figure below shows an annulus on the left and a Möbius band on the right. Each surface also has several curves γ_i ($i = 1, \dots, 4$) on them. Which of these are essential, which are separating?



2. If a surface consists of two discs and a single band joining them, it is homeomorphic to a single disc with no bands attached. Show by induction that any surface built by adding bands to a collection of discs can be built using only one disc.

3. Let F be a connected orientable surface formed by gluing m bands to n discs. Show that $2g(F) = 2 - n + m - |\partial F|$.

4. Show that for each n , there is a pretzel presentation of $C(-n, -2)$ which gives a genus-1 projection surface. (Hint: HW 1 might help.)

5. Show that the (p, q) torus knot is spanned by a projection surface of genus $1/2(p - 1)(q - 1)$.

6. Show that changing the orientation of some components in a link diagram can affect

the genus of the resulting projection surface.

7. *Class exercise.* Chapter 5 questions 6 and 7 (p. 127).

8. a) Show that companionship is reflexive and transitive.

b) Suppose that knots K_1 and K_2 are companions of each other. Show that they are equal up to orientation: $K_1 = K_2$ or $K_1 = -K_2$.

9. Show that a knot with no proper companions is prime.

10. Chapter 4 question 9 on Brunnian Links (p. 100).