Homework 2

Math 139

DUE: Thursday February 15, 2007.

Reference: Knots and Links Chapters 3 and 5.

1. Show that the invariant of oriented links $w(L)$ (writhe defined on page 62) is zero on all links.

2. Show that linking number is an integer.

3. Show that winding number is an integer.

4. Is there a diagram with writhe zero and winding number zero? Is the answer different for knots and links?

5. For this exercise you are only allowed to use the Reidemeister moves shown in Figure 3.17 page 68.

(a) Show how to get the mirror image of the type 1 move. (Hint: study Figure 3.22.)

(b) Show that the mirror image of the type 3 move can be obtained using a sequence of the given type 2 and 3 moves. (Hint: one of the intermediate steps may look like the picture on page 77.)

6. *Chessboard coloring.* Read question 13 on page 76 and make sure you think about why link projections have chessboard colorings. Now do question 14 on page 76. The point of this question is to show you that links can be the boundary of both orientable and non-orientable surfaces.

7. Draw projection surfaces (defined page 104) for pretzel knots. Show that the surface has genus 1 when all of p, q and r are odd. What happens when one of the coefficients is even?

8. Draw projection surfaces for the twist knots $C(-n, -2)$. Consider the cases when n is odd and n is even, and show that both give rise to genus-1 projection surfaces.