

Homework 1

Math 139

DUE: Thursday 8th February, 2007.

References: Knots and Links by Peter Cromwell Ch 1 and Ch 3.

1. Knots and Links page 30 question 3.
2. Prove that there are no nontrivial knots with polygon index 4 or 5.
3. (Class question) The knot 8_{17} is the first knot in the appendix that is not reversible, a difficult fact to prove. Find inversions for some of the knots that precede it. Several are not obvious.
4. An ascending diagram is one in which it is possible to choose a starting point and a direction such that each crossing is first encountered on the under-crossing strand. Show that an ascending diagram represents a trivial link, and deduce that any diagram can be converted into a diagram of a trivial link by switching crossings.
5. Show that $u(K) \leq \frac{1}{2}c(K)$.
6. The pretzel knot $P(-1, 3, 3)$ is a 7-crossing diagram of a much simpler knot. Identify it.
7. Show that, in order for $P(p, q, r)$ to be a knot, either all of p , q and r are odd, or exactly one of them is even. How many components are formed in the other cases?
8. Show that the twist knot $C(-n, -2)$ can be presented as a pretzel knot in two ways: $P(n, 1, 1)$ and $P(n - 1, -3, 1)$.

Optional problem: Prove that two polygonal knots K_1 and K_2 are combinatorially equivalent if and only if their knot diagrams are connected by a finite number of Reidemeister moves.