

# Syllabus for Math 136: Differential Geometry

Weiyang Qiu  
qiu@math.harvard.edu  
Science Center, 420  
www.fas.harvard.edu/~math136  
Office Hours: Mon 2-3PM, Wed 2-3Pm,  
or by appointment.

Sept. 2004

*Time and location:* MWF 11am, Science Center, room 411.

*Prerequisites:* Math 21a, b(Multi-variable calculus, linear algebra and differential equations) and familiarity with proofs as in Math 101, 112, 121, or equivalent.

This course will serve as an introduction to modern differential geometry. We will focus on the local and global theory of curves and surfaces in the three dimensional Euclidean space. We will also introduce Riemannian geometry on two-dimensional manifolds. Intrinsic and extrinsic properties of surfaces will be explored. Emphasis will be put on the fundamental concepts of Riemannian Geometry, such as geodesics, parallel translations, connections, curvatures and second fundamental forms. Also we will study the interaction between geometry and topology(e.g. Gauss-Bonnet Theorem). This course will be a good bridge to the study of more advanced Riemannian Geometry on higher dimensional abstract manifolds.

Here are materials that will be covered in this course:

1. *The local theory of curves:* Parametric representations; Arc length; Frenet formula; Existence and Uniqueness Theorem of curves.
2. *Plane curve theory:* Jordan curve theorem; winding number; convex curves; the Four-Vertex Theorem.
3. *The global theory of curves:* Total curvature; Deformations of curves.
4. *The local theory of surfaces:* Parametrizations; the first and second fundamental forms; Principal curvatures; the theorems of Dupin and Liouville; the fundamental equations and the fundamental theorem of surfaces; minimal surfaces; Levi-Civita parallelism; Geodesics.

5. *The global theory of Surfaces*: Orientation of surfaces; Surfaces with constant curvature; the Gauss-Bonnet formula.
6. *Two-dimensional Riemannian Geometry*: Manifolds; Riemannian metrics; tangent bundle and exponential maps; geodesics; Jacobi fields; first and second variational formula for curves;

Here are the textbook and reference:

- a. *Textbook*: Chuan-Chih Hsiung, *A First Course in Differential Geometry*, International Press, Cambridge, MA, 1997.
- b. *Reference*: M. P. do Carmo, *Differential Geometry of curves and surfaces*. Prentice-Hall, Englewood, NJ, 1976.

There will be weekly homework assignments, which will be posted on the course web-page. Homeworks are due each Wednesday. There will also be a midterm exam and a final exam, both of which will be take-homes. Homework will account for 30% of your final grade, midterm 30% and final exam 40%.