Solution for Homework #1 for Math 136

p. 82 #7

$(2, 2u, u)$; $(\sqrt{2}(1-u), \sqrt{2}(1+u), \pi/4 + u)$

p. 87 #6

$(\vec{q} - \vec{p}) \cdot \vec{u} = (\int_a^b \vec{x}'(t)dt) \cdot \vec{u} \leq \int_a^b ||\vec{x}'(t)||dt$. Plugging in $\vec{u} = (\vec{q} - \vec{p})/||\vec{q} - \vec{p}||$ gives the desired result.

p. 87 #7

The arc length arc length from $1/n$ to $1/(n+1)$ is given by $\int_{\frac{1}{(n+1)}}^{\frac{1}{n}} \sqrt{x_1'^2 + x_2'^2} \geq \int_{\frac{1}{(n+1)}}^{\frac{1}{n}} \sqrt{x_2'^2} = \frac{2}{n+1/2}$. The sum of $\frac{2}{n+1/2}$ over positive n diverges.

Additional Problem

In both cases we find that the function is constant. In the first, we use that the difference quotient of the derivative is zero for arbitrarily small τ. For the second we just note that the function is identical to $f(0)$ on some dense subset, and therefore is everywhere constant.