# Math 126, sheet 8

*April 21, 2000*

**Problem 1**. Find the (finite-dimensional, continuous) irreducible representations of the topological group $O(2)$, the group of orthogonal 2-by-2 matrices (which consists of rotations and reflections).

---

**Problem 2**. Let $G$ be a group, and let $A^{(i)}$ $(i = 1, 2, \ldots)$ be a sequence of matrix representations of the same degree:

$$A^{(i)} : G \to GL(n, \mathbf{C}).$$

Let us say that these representations *converge* to a matrix representation $A$ if $A^{(i)}(g) \to A(g)$ for all $g \in G$. (Note that the actual representations, rather than their equivalence classes, are involved in this definition.)

(a) If $G$ is a compact topological group and $A^{(i)}$ are continuous representations converging to $A$, show that $A^{(i)}$ is equivalent to $A$ for all sufficiently large $i$.

(b) Show that the corresponding statement may be false if $G$ is merely an infinite group.

---

**Problem 3**. Let $G$ be a group and let $A^{(i)}$ be representations of $G$ converging to $A$ in the sense discussed above.

If each $A^{(i)}$ is reducible, show that $A$ is reducible. Give an example to show the converse is false.

If each $A^{(i)}$ is *completely reducible* (i.e. equivalent to a direct sum of reducibles), does it follow that $A$ is completely reducible?

---

*Peter Kronheimer*