Solution Set 6

No one had trouble with 1, 3, or 5.

2. One efficient way to do this problem is the following. Observe that there are at *least* 166 elements whose square is 1. Next, given that the indicators of $\chi_3, \chi_4, \chi_6, \chi_7$ must be zero and that the others must be ± 1, we see that $\sum \mathrm{ind}(\chi_i)\chi_i(1)$ is at *most* 166. It follows that the indicators of the remaining characters are all 1.

We then immediately see that there is only one conjugacy class of order 2 elements. Furthermore, for any g of order 2, the number of elements whose square is g is $\sum \mathrm{ind}(\chi_i)\chi_i(g) = 6$.

4.

$$\begin{aligned}
a_{ij}b_{pq} &= \sum_{g,h\in G} a_{ij}(g)b_{pq}(h)gh \\
&= \sum_{g,k\in G} a_{ij}(g)b_{pq}(g^{-1}k)k \\
&= \sum_{g,k\in G}\sum_{l=1}^{d} a_{ij}(g)b_{pd}(g^{-1})b_{dq}(k)k \\
&= \sum_{k\in G}\sum_{l=1}^{d}\left(\sum_{g\in G} a_{ij}(g)b_{pd}(g^{-1})\right) b_{dq}(k)k \\
&= 0
\end{aligned}$$

since the quantity in the parentheses vanishes by orthogonality. A similar argument works for the other relation.