Math 126, sheet 6

April 7, 2000

Problem 1. Find the indicator functions for the characters of the groups S_4 and S_5.

Problem 2. Here is the character table of the Mathieu group, M_{11}.

	(1)	(165)	(440)	(990)	(1584)	(1320)	(990)	(990)	(720)	(720)
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	10	2	1	2	0	-1	0	0	-1	-1
χ_3	10	-2	1	0	0	1	α	$\bar{\alpha}$	-1	-1
χ_4	10	-2	1	0	0	1	$\bar{\alpha}$	α	-1	-1
χ_5	11	3	2	-1	1	0	-1	-1	0	0
χ_6	16	0	-2	0	1	0	0	0	β	$\bar{\beta}$
χ_7	16	0	-2	0	1	0	0	0	$\bar{\beta}$	β
χ_8	44	4	-1	0	-1	1	0	0	0	0
χ_9	45	-3	0	1	0	0	-1	-1	1	1
χ_{10}	55	-1	1	-1	0	-1	1	1	0	0

Here $\alpha = i\sqrt{2}$ and $\beta = (-1 + i\sqrt{11})/2$. The numbers in parentheses at the top are the size of the conjugacy classes. Find the indicator function for this group.

Show there is only one conjugacy class of elements of order 2, and that for each such element g there are exactly 6 elements x of order 4 with $x^2 = g$.

Problem 3. Show that for all $n \geq 4$, the symmetric group S_n has at least two irreducible characters of degree $n - 1$, and that for $n = 6$ it has 4.

Problem 4. For each representation $A(g) = \big(a_{ij}(g)\big)$ of degree d, define d^2 elements of the group algebra $R = \mathbf{C}G$ by $a_{ij} = \sum_g a_{ij}(g)g$. Show that the orthogonality relations can be stated as follows. For irreducible representations A, B that are inequivalent, $a_{ij}b_{pq} = 0$ in R, while $a_{ij}a_{pq} = (|G|/d)\delta_{jp}iq$.

Problem 5. Find the Young diagrams that correspond to the irreducible representation(s) of S_6 of largest degree.

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