# Math 126, sheet 1

*February 11, 2000*

**Problem 1.** Let $V$ be a G-module over any field $K$, and let $v \in V$ be any element. If $G$ is finite, show that there is a submodule $U \subseteq V$ containing $v$ with $\dim U \leq |G|$.

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**Problem 2.** Show that if $G$ is a finite group and $|G| > 1$, then any irreducible $G$-module (over any field $K$) has dimension strictly less than $|G|$.

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**Problem 3.** Use the 'Corollary to Schur's Lemma' to show that every irreducible complex representation of a finite *abelian* group is 1-dimensional. Deduce that a complex matrix $A$ satisfying $A^k = I$ is diagonalizable.

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**Problem 4.** Let $D_{2k}$ be the dihedral group of order $2k$. We can think of $D_{2k}$ as the symmetry group of a regular $k$-gon. As such, it is generated by two elements: $x$, a rotation through angle $2\pi/k$, and $y$, a reflection. These satisfy the relations $x^k = 1$ , $y^2 = 1$ and $yxy = x^{-1}$.

Classify the 1-dimensional complex representations of $D_{2k}$. (The cases $k$ even and $k$ odd are different!)

Write down $\lfloor (k-1)/2 \rfloor$ irreducible complex matrix representations of degree 2 and show that they are inequivalent.

Show by direct arguments that these 1- and 2-dimensional representations exhaust all the irreducible complex representations of $D_{2k}$. Then verify for $D_{2k}$ the Theorem stated in class (as yet unproved): the number of irreducible complex representations is equal to the number of conjugacy classes of the group, and the sum of the squares of their degrees is equal to the order of the group.

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**Problem 5**. Use the corollary to Schur's Lemma to show that if $G$ is a finite group which has a *faithful*, irreducible complex representation, then the center of $G$ is cyclic.

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