

Math 126, Problem Set 2

This problem set is due October 9, 1998.

1. (25 pts.) Let R be a semisimple ring. Let M be an R -module with $M \neq 0$. Using the notation from class, write $R = \prod_{i=1}^m R_i$, where R_i is a simple ring.

Show that

$$M = \bigoplus_{i=1}^m R_i M = \bigoplus_{i=1}^m e_i M,$$

where $1 = \sum_{i=1}^m e_i$ and $e_i \in R_i$.

Show that

$$R_i M = \sum_{N \cong L_i} N,$$

where L_i is a simple left ideal in R_i .

2. (25 pts.) Are groups, up to isomorphism, uniquely determined by their character table? Hint: Compute the character tables for the groups O and D_4 , where D_4 is the dihedral group of order 8 and O is the quaternion group. That is, $O = \{\pm 1, \pm i, \pm j, \pm k\}$ where $i^2 = j^2 = k^2 = -1$ and $ijk = -1$.

3. (25 pts.) Compute the character table for S_4 .

4. (25 pts.) For each n , construct all the simple one and two dimensional representations of the symmetric group S_n .