In this course we will study representations $G \to Aut(V)$ of finite groups, G is a finite group and V a finite-dimensional complex vector space. I assume that you have a reasonable knowledge of linear algebra and some familiarity with the basis concepts of the group theory. 1. We start with basic definitions of the group theory. 2. Then we discuss basic definitions of the representation theory such as **representations, equivalence of representations, irreducibilty** and consider a couple of examples. In particular we discuss **the regular representation** R(G) for a finite group G. 3. We prove two BIG theorems in the theory of representations of finite groups

A) the complete irreducibility of representations.

B)The Schur's lemma. 4. From these theorems we derive two important consequences

A). The number of equivalence classes of irreducible representations of a finite group G is equal to the number of conjugacy classes of G and

B) The sum of squares of dimensions of irreducible representations of a finite group G is equal to the order of G. 5) We consider three ways of constructions of representations

a)The restrictions

b) The induction

c)The decomposition into irreducible summands. 6) We develop the theory of **characters** for finite-dimensional representation, prove orthogonality relations for characters and find characters of irreducible representations for a number of "easy" groups. 7) We apply the theory of characters to problems in representation theory (to show that the dimension of any irreducible representation of a group G divides the order of G) and to the group theory (to show that any finite group of order $p^a q^b$ [p,q are primes] is solvable). 8) We study in details the description of irreducible representation of symmetric groups S_n and the characters of irreducible representation of S_n. 9) We study irreducible representations of the group $GL(2, \mathcal{F}_q)$ of 2×2 matrices over a finite field $\mathcal{F}_q$ and the characters of irreducible representation of the group $GL(2, \mathcal{F}_q)$.

The course will be given on Tue-Th 10-11.30.

We will NOT have classes on Tuesdays September 18-th, October 2-d and October 9-th and on Thursday September 27-th [these are days of Jewish holidays]

I'll assign homework problems every week and we will have one mid-term exam.