MATH 126 PROBLEM SET 2

Due on Tuesday Sep.25-th

If $\rho' : G \to Aut(V'), \rho'' : G \to Aut(V'')$ are representations we denote by $Hom_G(V', V'')$ the space of linear maps $T : V' \to V''$ such that $\rho''(g) \circ T = T \circ \rho'(g)$ for all $g \in G$

If a group G acts on a set X,$(g, x) \to gx$ we denote by $\mathbb{C}(X)$ the space of functions on X and by ρ_X the representation of G on $\mathbb{C}(X)$ given by $\rho_X(g)(f)(x) := f(g^{-1}x)$

1. Let X_F be the set of six faces of a cube in 3-dimensional space, G the group of rotations of the cube,L_F the space of functions on X_F and $\rho_F : G \to Aut(L_F)$ the representation given by $\rho_F(g)(f)(x) := f(g^{-1}x)$

a) Find the dimension of the space $End_G(L_F)$

b) How many irreducible subrepresentations are in L_F?

c) Describe all irreducible subrepresentations of L_F.

d) Let $T : L_F \to L_F$ be a linear operator such that $T(f)(x) := 1/4 \sum f(y)$ where the summation is extended to the four faces y that are adjacent to x. Let $l \in L_F$ be a function which takes six different values $1, 2, 3, 4, 5, 6$ on six elements of X_F. Prove that for any $x \in X_F$ we have $|T^{30}l(x) - 3.5| < 10^{-8}$.

Let V,M',M" be vector spaces,$V' := M' \otimes V, V'' := M'' \otimes V$

2. a) Show that for any linear map $T \in Hom(M', M'')$ there exists unique linear map $\alpha(T) \in Hom(V', V'')$ such that $\alpha(T)(m' \otimes v) = T(m') \otimes v$ for all $m' \in M', v \in V$

Let $\rho : G \to Aut(V)$ be a representation of a group G on V,$\rho' : G \to Aut(V'), \rho'' : G \to Aut(V'')$ be representations given by

$$\rho'(g)(m' \otimes v) := m' \otimes \rho(g)(v), \rho''(g)(m'' \otimes v) := m'' \otimes \rho(g)(v), m' \in M', m'' \in M'', v \in V$$

b)Show that for any $T \in Hom(M', M'')$ we have $\alpha(T) \in Hom_G(V', V'')$ [that is $\rho''(g) \circ \alpha(T) = \alpha(T) \circ \rho'(g)$ for all $g \in G$],

c)Show that in the case when ρ is an irreducible representation the map $\alpha : Hom(M', M'') \to Hom_G(V', V'')$ is one-to-one and onto [in other words the map $\alpha : Hom(M', M'') \to Hom_G(V', V'')$ is an isomorphism of linear spaces.

3. Let $\rho' : G \to Aut(V'), \rho'' : G \to Aut(V'')$ be irreducible NON EQUIVALENT representations. Show that $Hom_G(V', V'') = \{0\}$