

# Homework 9

Math 124, Fall 2005

Due Wednesday, November 23rd

No late assignments will be accepted.

1. How many elliptic curves are there modulo 5? How many of these are non-singular? Of the non-singular curves, how many inequivalent ones are there?
2. Find all the rational points of the equation

$$y^2 = x^3 - 4x. \tag{1}$$

- (a) Show that rational points of (1) are on a line  $y = mx$  where  $m$  is rational.
- (b) Show that if the line  $y = mx$  meets the elliptic curve of (1) at three rational points then there exists a rational  $n$  such that

$$m^4 + 16 = n^2.$$

- (c) Prove that the equation

$$U^4 + W^4 = V^2W^2$$

has only trivial solutions over the integers. [Hint : follow the proof that  $x^4 + y^4 = z^2$  has no solutions.]

- (d) Find all possible values of  $m$  and find all rational points of (1).

3. Say  $P = (x, y)$  is a point on the curve  $y^2 = x^3 - Ax - B$  with  $y \neq 0$ . Use the geometric construction to prove the formula of page 162 for  $2P = (a, b)$ .

$$\begin{aligned} a &= \left( \frac{3x^2 - A}{2y} \right) - 2x \\ b &= \left( \frac{3x^2 - A}{2y} \right) (x - a) - y. \end{aligned}$$