

# Homework 5

Math 124, Fall 2005

Due Wednesday, October 26

No late assignments will be accepted as solutions will be posted on Thursday morning.

1. Express the following numbers as sums of two squares or prove that it is not possible. Show all your work.

97 221 300 490 729 1001 490490

2. Use Serret's construction to write 449 as a sum of two squares. [Hint :  $67^2 \equiv -1 \pmod{449}$ .]
3. (a) Prove that the following equation has a solution.

$$h^2 \equiv -5 \pmod{23}$$

- (b) Prove that the following equation has no solutions.

$$23 = x^2 + 5y^2$$

- (c) Conclude that there are more than one equivalence classes of quadratic forms of discriminant -5.
4. (a) Prove that there are two equivalence classes of quadratic forms of discriminant -12, corresponding to the following two reduced forms.

$$x^2 + 3y^2 \quad 2x^2 + 2xy + 2y^2$$

- (b) Show that a prime  $p$  is representable by  $x^2 + 3y^2$  if and only if  $p = 2$  or  $p \equiv 2 \pmod{6}$ .