Homework 4

Math 124, Fall 2004

Due Wednesday, October 19

No late assignments will be accepted as solutions will be posted on Thursday morning. Good luck!

Throughout this assignment we will denote by

$$< q_0, q_1, \dots q_n >= q_0 + \cfrac{1}{q_1 + \frac{1}{q_2 + \frac{1}{q_3 + \dots}}}$$

1. Find all primes p such that the equation

$$x^2 \equiv 13 \mod p$$

   has a solution.

2. (a) Evaluate the following infinite continued fractions.
      i. $< 2, 3, 1, 1, 1, 1, 1, \cdots >$
      ii. $< 1, 3, 1, 2, 1, 2, 1, 2, \cdots >$
   (b) Expand $\sqrt{5}$ and $\sqrt{20}$ as continued fractions.
   (c) Find the continued fraction for the number $\sqrt{n(n+1)}$.

3. (a) Given positive integers $d < c$ show that

$$\begin{aligned} < a, c > \quad &< \quad < a, d > \\ < a, b, c > \quad &> \quad < a, b, d > \end{aligned}$$

   (b) Let $a_1, \dots a_n$ and c be positive real numbers. Prove that

$$\begin{aligned} < a_0, a_1, \dots, a_n + c > \quad &< \quad < a_0, \dots a_n > \qquad n = 2k+1 \\ < a_0, a_1, \dots, a_n + c > \quad &> \quad < a_0, \dots a_n > \qquad n = 2k \end{aligned}$$

   (c) Let $a_0, \dots a_n$ and $b_0, \dots b_{n+1}$ be positve integers. State the conditions for

$$< a_0, \dots, a_n > \quad < \quad < b_0, \dots, b_n, b_{n+1} > .$$