math123, Abstract Algebra II

PROBLEM SET 9

- Exercise 13.5.1 from Artin's book
- Exercise 13.5.2
- Exercise 13.6.5
- Exercise 13.6.7
- Exercise 13.6.8
- Exercise 13.6.10
- Exercise 13.6.13
- Exercise 13 Misc1
- Exercise 13 Misc2
- Exercise 13 Misc5
- Solve the following

  Problem 1:

  Let p be a prime. Denote by $q_n = p^{n!}$ and by K_n the field with q_n elements. We then have a sequence of field extensions:

  $$\mathbb{F}_p = K_1 \subset K_2 \subset \cdots \subset K_n \subset \cdots$$

  Prove that the union:

  $$\bar{\mathbb{F}}_p \;=\; \bigcup_{n\geq 1} K_n$$

  is an algebraically closed field of characteristic p, and that every finite field K of characteristic p is embedded in $\bar{\mathbb{F}}_p$.

  Remarks: This is stated in Section 9 of Artin's book. I ask you to fill in the details of the proof.

  Notice that this proves that every finite field is embedded in an algebraically closed field. The same result is actually true for every field $\mathbb{F}$. The proof in the general case is not very hard and, if you are interested, you can find a nice proof of this statement in the book of Lang, page 273 (Unfortunately, I won't have time to cover it in class).