Abstract Algebra Study Sheet

This study sheet should serve as a guide to which sections of Artin will be most relevant to the final exam. When you study, you may find it productive to prioritize the definitions, examples, results and exercises with which you are least comfortable and which are most likely to be important on the final exam.

2.1. *Definition of group.*
You should know all the definitions and results in this section cold.

2.2. *Subgroups.*
Again, the definitions, results and examples here should be like second nature.

2.3. *Isomorphisms.*
The idea of identifying isomorphic groups should be completely familiar.

2.4. *Homomorphisms.*
Make sure you can easily identify familiar examples of homomorphisms, and that you have good facility for calculating image and kernel. Also make sure you can identify normal subgroups of familiar examples of groups.

2.5. *Equivalence relations and partitions.*
The ideas here should now be easy because you have practiced with the coset space construction.

2.6. *Cosets.*
The results here are very important. Being able to detect equality of cosets, knowing the relationship between the order of an element and the order of a group, and knowing the combinatorics of kernel and image are very important.

2.7. *Restriction of homomorphisms.*
These results should be straightforward and familiar.

2.8. *Products of groups.*
You should know how to work with products of groups.

2.9. *Modular arithmetic.*
Make sure to have a very strong ability to calculate in modular arithmetic.

2.10. *Quotient groups.*
The relationship between quotient groups and normal subgroups should be entirely intuitive. The first isomorphism theorem is a very important tool.

3.1. *Real vectorspaces.*
This is elementary.

3.2. *Abstract fields.*
Now that you know all about rings, this material should be easy. Make sure you are comfortable calculating in abstract fields, with special attention to fields in which $p = 0$.

3.3. *Bases and dimension.*
Make sure you know the concepts linear combination, span, linear independence, basis and dimension. The relationship between different bases of a fixed vectorspace is important.

3.4. *Computation with bases.*
You should know how ordered bases correspond to invertible matrices.

3.6. *Direct sums.*
You should have a general sense of the results here.

4.1. *Dimension formula.*
This should be familiar as an analogue of the group theoretic combinatorics of image and kernel.

4.2. *Matrix of a linear transformation.*
Make sure you have a good feel for the relationship between matrices and linear transformations equipped with an ordered basis.

4.3. *Linear operators and eigenvectors.*
This should be familiar from previous linear algebra courses. You should understand the notion of similarity of matrices, and its relationship with change of basis.

4.4. *Characteristic polynomial.*
Make sure you understand well the concept of characteristic polynomial and the fact that it is invariant under change of basis.

4.5. *Orthogonal matrices.*
You should have a solid understanding of the terms orthogonal matrix and orthonormal subset of $\mathbb{R}^n$ (i.e set of mutually orthogonal vectors of length 1), and you should know the relationship between the two. You should be very comfortable with the results in this section.

5.1. *Symmetry of plane figures.*
Motivational.

5.2. *Group of motions of the plane.*
You should remember concepts like motions, orientation, translations, rotations, reflections; and the statements of the structure theorems that relate them.

5.3. *Finite groups of motions.*
You should recall the statements of the classification results.

5.4. *Discrete groups of motions.*
You should recall the notion of discrete group of motions and the statements of the classification results. Also you should recall lattices in $\mathbb{R}^2$ and their structure theory.

5.5. *Abstract symmetry: group actions.*
This is very important material. Make sure you have a good memory of what a group action is, and concepts like transitive action, stabilizer, orbit. You should have a strong intuition for group actions.

5.6. *Operation on cosets.*
You should have a solid understanding of the coset space G/H as a G-set, and the relationship between orbit and stabilizer.

5.7. *Counting formula.*
You should be comfortable with the combinatorics here and how it gets applied in familiar examples.

5.8. *Permutation represenations.*
You should be comfortable with the correspondence between the actions of a group G on a set S and homomorphisms $G \to \mathrm{Sym}(S)$.

6.1. *Operations of a group on itself.*
The definitions, examples and results in the section are all extremely important.

6.2. *Class equation of the icosahedral group.*
This example is a good check on how well you understood the previous section, but will not be specifically emphasized on the exam.

6.3. *Operations on subsets.*
The concepts of conjugate subgroups and normalizer are very important.

6.4. *Sylow theorems.*
The entirety of this section is indispensable.

6.5. *Groups of order 12.*
This example is a good check on how well you understood the previous section, but will not be specifically emphasized on the exam.

6.6. *Computations in the symmetric group.*
This section is extremely important, especially the characterization of conjugacy in the symmetric group and how that relates to the rest of the chapter, as well as the Sylow p-subgroups of S_p.

10.1. *Definition of a ring.*
You should know all the definitions and results in this section cold.

10.3. *Homomorphisms and ideals.*
Again, the definitions, results and examples here should be like second nature.

10.4. *Quotient rings and relations in a ring.*
This entire section is very important.

10.5. *Adjunction of elements.*
This is a very useful section. Adjunction examples like $\mathbb{Q}(\sqrt{d})$ and $\mathbb{Z}[(1+\sqrt{d})/2]$ should be completely familiar, as should more exotic examples like adjoining the inverse of an element of a ring: $R[x^{-1}]$ where $x \in R$ is not a zero divisor.

10.6. *Integral domains.*
The definitions here are very important, as is the construction of the field of fractions.

10.7. *Maximal ideals.*
You should know the definitions and results from the first part of the section; you do not need to know the Nullstellensatz.

11.1. *Factorization of integers and polynomials.*
You should know the examples here.

11.2. *UFDs, PIDs, and euclidean domains.*
The definitions, results and examples here are very important.

11.3. *Gauss' lemma.*
You should know Gauss' lemma cold and how to use it.

11.4. *Explicit factorization of polynomials.*
You should review the techniques here.

11.5. *Primes in the ring of Gauss integers.*
You should be comfortable with the structure theory here.

11.6. *Algebraic integers.*
You should be able to identify the ring of all algebraic integers in a quadratic field and you should have an intuition for the arithmetic of such a ring.

11.7. *Factorization in imaginary quadratic fields.*
You should have a solid understanding of the units in the ring of algebraic integers in a quadratic field, and you should have an intuition for factorization.

11.8. *Ideal factorization.*
You should know concepts like product of ideals, and the big picture of factoring ideals in rings of algebraic integers.

11.9. *Factoring prime ideals.*
You should make sure you really understand this section.

11.10. *Ideal classes in imaginary quadratic fields.*
Not on the exam.