$\boxed{5.1.14}$ $A \in M_{n\times n}(F) \Rightarrow$ ~~the~~ char. poly. of $A$ = char. poly. of $A^T$.

From Thm. 4.8, we know that $\det(B) = \det(B^T)$, $\forall B \in M_{n\times n}(F)$ $\Big| \Rightarrow$

Char. poly. of $A$ is, by defn, $\det(A - tI)$. Let $B = A - tI$

$$\Rightarrow \det(A - tI) = \det(B) = \det(B^T) = \det[(A-tI)^t] =$$
$$= \det(A^T - tI^t) \quad \text{(by properties of transpose)}$$
$$= \det(A^T - tI), \text{ which is also the}$$

$\boxed{5.1.17}$ $T$ linear op on $M_{n\times n}(R)$, $T(A) = A^t$

(a) Show that $\pm 1$ are the only eigenvalues of $T$.

Suppose $\lambda$ is an eigenvalue of $T \Rightarrow T(A) = \lambda A$.

Look at $\left\{\begin{array}{l} T(T(A)) = T(\lambda A) = \lambda^2 A \\ T(T(A)) = T(A^t) = (A^t)^t = A \end{array}\right\} \Rightarrow \lambda^2 A = A \Leftrightarrow (\lambda^2 - 1)A = 0, \forall A$

on the other hand $\Leftrightarrow \underline{\underline{\lambda = \pm 1}}$.

(b) $\lambda = 1 \Leftrightarrow T(A) = A^T = A \Rightarrow$ set of symmetric matrices.

$\lambda = -1 \Leftrightarrow T(A) = -A^T = A \Rightarrow$ set of antisymmetric matrices.

(c) $\beta = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\} \Rightarrow [T]_\beta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

(d) Let $\beta = \{A_i\} \cup \{B_{jk}\} \cup \{C_{lm}\}$ where:

$A_i$ : coeff $a_{ii} = 1$, $a_{ij} = 0$, $\forall i \neq j$ for $1 \le i,j \le n$.

$B_{jk}$ : coeff. $b_{jk} = b_{kj} = 1$, all other entries are 0.

$C_{lm}$ : coeffs. $c_{lm} = 1$, $c_{ml} = -1$, all other entries are 0.

So $[T]_\beta$ looks like:

$$\left[\begin{array}{ccc|ccc} 1 & & 0 & & & \\ & \ddots & & & 0 & \\ 0 & & 1 & & & \\ \hline & & & -1 & & 0 \\ & 0 & & & \ddots & \\ & & & 0 & & -1 \end{array}\right] \begin{array}{l} \frac{n^2+n}{2} \\ \\ \frac{n^2-n}{2} \end{array}$$