MATH 121 SIMONA TOPOR POP

# HOMEWORK #9 SOLUTIONS

## 5.1.1

(a) False. Eg The identity matrix

(b) True.

(c) True. Eg: The zero matrix

(d) False. ~~Eg: It could be zero~~. Eg. Can't have zero as only eigenvector

(e) False. Eg.: They could correspond to the same eigenvalue.

(f) False. There are counter examples.

(g) False. Eg: The derivative map w/ $e^x$ as eigenvector

(h) True.

(i) True. Sure - they have the same det.

(j) False. Remember that you can have many eigenvectors assoc. to an eigenvalue

(k) False. There are many examples

## 5.1.3

(c) $A = \begin{bmatrix} i & 1 \\ 2 & -i \end{bmatrix}$, $\mathbb{F} = \mathbb{C}$

(i) $f(t) = t^2 - 1 \Rightarrow \lambda_1 = 1$, $\lambda_2 = -1$

(ii) $\lambda_1 = 1 \longleftrightarrow \begin{pmatrix} 1 \\ 1-i \end{pmatrix} t$, $t \in \mathbb{C}^*$

$\lambda_2 = -1 \longleftrightarrow \begin{pmatrix} 1 \\ -1-i \end{pmatrix} t$, $t \in \mathbb{C}^*$

Note: There are many equivalent answers that turn out to be multiples of these. As long as you check that your answer verifies $A\vec{x} = \lambda\vec{x}$, you should be fine.

(iii) $\left\{ \begin{pmatrix} 1 \\ 1-i \end{pmatrix}, \begin{pmatrix} 1 \\ -1-i \end{pmatrix} \right\}$ — linearly independent, so it's a basis for $\mathbb{F}^2$.

(Once again, there are many equivalent answers satisfying this)