

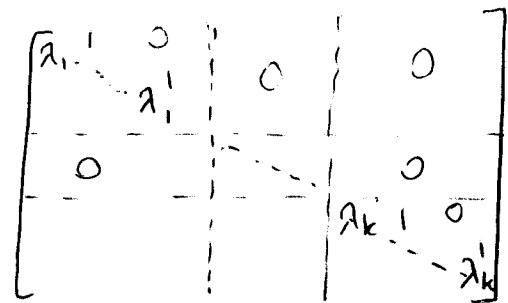
So $\exists Q, S$ invertible matrices s.t. $Q^{-1}AQ = J = S^{-1}A^t S$

$$\Rightarrow A = (QS^{-1})A^t(SQ^{-1}) \Rightarrow$$

$$\Rightarrow A = T^{-1}A^t T$$

Hence ~~also~~ A, A^t are similar matrices. Q.E.D.

7.28 (Thanks to Barbara Richter)

(a) Take $\beta = \{x_1, \dots, x_n\}$, $[T]_\beta =$ 

β is a Jordan canonical basis, so we either have:

$$T(x_i) = \lambda x_i \text{ or } T(x_i) = x_{i-1} + \lambda x_i \text{ for some eigenvalue } \lambda.$$

Consider $\beta' = \{v_1, \dots, v_n\}$, $v_j = c x_j$, $j = \overline{1, n}$.

$$\Rightarrow \begin{cases} \underline{T(v_i)} = T(c x_i) = c T(x_i) = c(\lambda x_i) = \lambda(c x_i) = \underline{\lambda v_i} \text{ or:} \\ \underline{T(v_i)} = T(c x_i) = c T(x_i) = c(x_{i-1} + \lambda x_i) = c x_{i-1} + \lambda(c x_i) = \\ = \underline{v_{i-1} + \lambda v_i} \end{cases}$$

So β' is also a Jordan canonical basis for T .

(b) $\beta = \{(T-\lambda I)^k(x), \dots, (T-\lambda I)(x), x\}$.

$$\beta' = \{(T-\lambda I)^k(x+y), \dots, (T-\lambda I)(x+y), x+y\}$$

First prove β' is cycle of gen. eigenvect. corresp to λ :

$$\begin{matrix} y \in N(T-\lambda I) \subseteq N((T-\lambda I)^{k+1}) \\ x \in N((T-\lambda I)^{k+1}) \end{matrix} \Rightarrow x+y \in N((T-\lambda I)^{k+1})$$

Now prove that replacing β by β' in defn of β , we end up w/ Jordan canonical basis:

$$(T-\lambda I)(x+y) = (T-\lambda I)x + (T-\lambda I)y = (T-\lambda I)x \quad (\text{why is this?})$$

$$\text{So: } \beta' = \{(T-\lambda I)^k(x) \dots (T-\lambda I)(x), x+y\}$$

Know: $T(x) = (T-\lambda I)x + \lambda x$. So just check: $T(x+y) = (T-\lambda I)x + \lambda(x+y)$ which works, so we are done!

(c) Ex. $\beta' = \left\{ \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$
 or $\begin{pmatrix} 0 \\ 1 \\ 2 \\ 0 \end{pmatrix}$