So: $Q = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & -1 \\ 3 & -1 & 1 \end{bmatrix} \rightsquigarrow J = Q^{-1}AQ = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

★ Several answers are possible for this – so if your answer didn't look exactly like this, I didn't take off any points ~ as long as you ended up with an ~~similar~~ equivalent canonical form!

$\boxed{7.2.5}$ (a) $\{e^t, te^t, t^2e^t, e^{2t}\}$, $T(f) = f'$.

Jordan canonical form: $J = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

Jordan canonical basis for T: $\beta = \{2e^t, 2te^t, t^2e^t, e^{2t}\}$.

(d) $M_{2\times 2}(\mathbb{R})$, $T(A) = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}(A - A^t)$

Jordan canonical form:

$J = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

Jordan canonical basis for T: $\beta = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix} \right\}$

$\boxed{7.2.6}$ Proved on HW. 9 that A & A^t have the same characteristic polynomial & the same eigenvalues.

Also, $\dim(N(A-\lambda I)^k) = \dim(N((A-\lambda I)^t)^k) = \dim(N(A^t - \lambda I)^k)$

Look at K_λ for a typical eigenspace which has Jordan block:

$\begin{bmatrix} \lambda & 1 & & \text{etc.} \\ & \lambda & 1 & \\ & & \lambda & \\ & & & \ddots \; \lambda \end{bmatrix}$ – from this, we know Null. $(A-\lambda I)^j$, and hence rank $(A-\lambda I)^j$.

But, by dim. formula, $\text{rank}(A-\lambda I)^j = \text{rank}\,[((A-\lambda I)^j)^t] = \text{rank}[(A-\lambda I)^t]^j = \text{rank}\,(A^T - \lambda I)^j$

So the Jordan blocks correspond exactly from A to A^T, and hence the Jordan forms of A and A^T must be the same.